UCLA AP Readiness APRIL 2018 Checklist of Skills and Mixed Review 2000 and 1999 Exam FRQS

I. Functions, Graphs, and Limits

Limits of functions (including one-sided limits) …

- An intuitive understanding of the limiting process
- Calculate limits using algebra
- Estimate limits from graphs or tables of data

Asymptotic and unbounded behavior …

- Understand asymptotes in terms of graphical behavior
- Describing asymptotic behavior in terms of limits involving infinity
- Compare relative magnitudes of functions and their rates of change

Continuity as a property of functions …

- An intuitive understanding of continuity
- Understand continuity in terms of limits
- Geometric understanding of graphs of continuous functions (**I**ntermediate **V**alue **T**heorem)

II. Derivatives

Concept of the derivative …

- Derivative presented graphically, numerically, and analytically
- Derivative interpreted as an instantaneous rate of change
- Derivative defined as the limit of the difference quotient
- Relationship between differentiability and continuity

Derivative at a point …

- Slope of a curve at a point. Know when a derivative does not exist.
- Tangent line to a curve at a point and local linear approximation
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values

Computation of derivatives …

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- Derivative rules for sums, products, and quotients of functions
- Chain rule and implicit differentiation

Derivative as a function …

- Know corresponding characteristics of graphs of f and f'
- Know relationship between the increasing and decreasing behavior of f and the sign of f'
- The **M**ean **V**alue **T**heorem and its geometric interpretation

Second derivatives …

- Know corresponding characteristics of the graphs of f, f', and f''
- Know relationship between the concavity of f and the sign of f''
- Points of inflection as places where concavity changes

Applications of derivatives …

- Analysis of curves, including the notions of monotonicity and concavity …
- Optimization, both absolute (global) and relative (local) extremes …
- Modeling rates of change, including related rates problems …
- Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration …
- Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations …
- Use L'Hopital's rule to find limits with indeterminate forms

III. Integrals

Numerical approximations to definite integrals …

- Use of Riemann sums (Left, Right, and Midpoint evaluation points) and Trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

Interpretations and properties of definite integrals …

- Definite integral as a limit of Riemann sums
- Basic properties of definite integrals (examples include additivity and linearity)

Fundamental Theorem of Calculus …

- Use of the Fundamental Theorem to evaluate definite integrals
- Definite integral of the rate of change of a quantity interpreted as the net change in that quantity over the interval: $\int_a^b f(x)dx = f(b) - f(a)$ or $f(a) + \int_a^b f(x)dx = f(b)$

Techniques of Integration (anti-differentiation) …

- Anti-derivatives following directly from derivatives of basic functions
- Anti-derivatives by u-substitution of variables (including change of limits for definite integrals)

Applications of antidifferentiation …

- Find specific anti-derivative general solutions and particular solutions
- Solve separable differential equations and write the particular solution as a function.

Applications of integrals …

- Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. The emphasis is on setting up a definite integral. Specific applications:
  - to find the area of a region
  - the volume of a solid with known cross sections
  - the average value of a function …
  - the distance traveled by a particle along a line
  - accumulated change from a rate of change

CALCULATOR functions:

- graph a function and adjust the window appropriately for analysis
- find specific function values, zeros, & intersection points
- Find the numerical derivative (at a point on a curve & as a graph)
  - **nderiv (** ***f (x)* , *x*, *a*** **)** to find *f '(a)* at *x = a*
  - **Y1 = nderiv(** ***f (x)*, *x*, *x*** **)** to graph the numerical derivative function *f '(x)*
- Find the numerical definite integral **fnInt (** ***f (x)*, *x*, *a*, *b*** **)**

PRIOR KNOWLEDGE (Just about everything from your high school career):

- Trigonometry function definitions and values from the Unit Circle

Parent Functions (properties, domains, ranges): $y = x^n$, $y = \sqrt[n]{x}$, $y = |x|$,

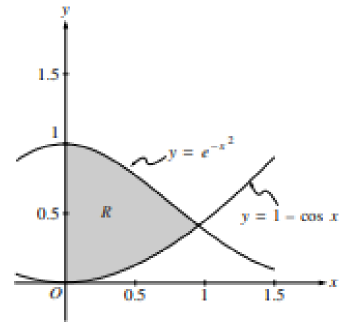
$y = e^x$, $y = \ln x$, $y = \sin x$, $y = \cos x$, $y = \tan x$

- Geometry Areas: Rectangles, Triangles, Trapezoids, Circles, plus

Algebra: factoring, domain restrictions, exponent rules, inverse operations

UCLA AP READINESS FRQ PRACTICE- Comprehensive Review

2000 #1



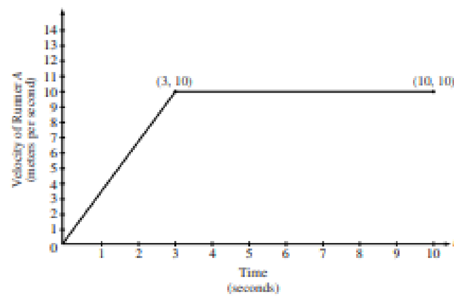
Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

2000 #2



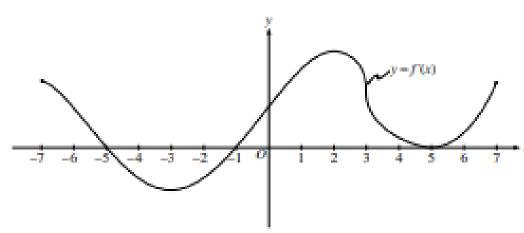
Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.

(a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

(b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

2000 #3



The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

(a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

(c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

(d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

2000 #4

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

2000 #5

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

2000 #6

Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

1999 #3

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24–hour period.

(a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

(b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

(c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24–hour time period. Indicate units of measure.

1999 #4

Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.

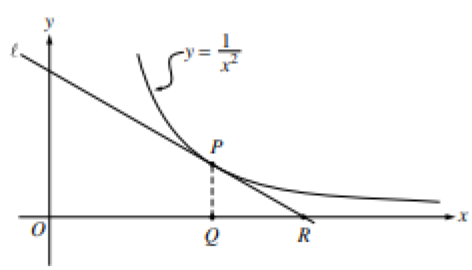
(a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.

(b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.

(c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.

(d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

1999 #6



. In the figure above, line ℓ is tangent to the graph of $y = \frac{1}{x^2}$ at point P, with coordinates $\left(w, \frac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x–axis at the point R, with coordinates $(k, 0)$.

(a) Find the value of k when $w = 3$.

(b) For all $w > 0$, find k in terms of w.

(c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

(d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of ΔPQR with respect to time? Determine whether the area is increasing or decreasing at this instant.