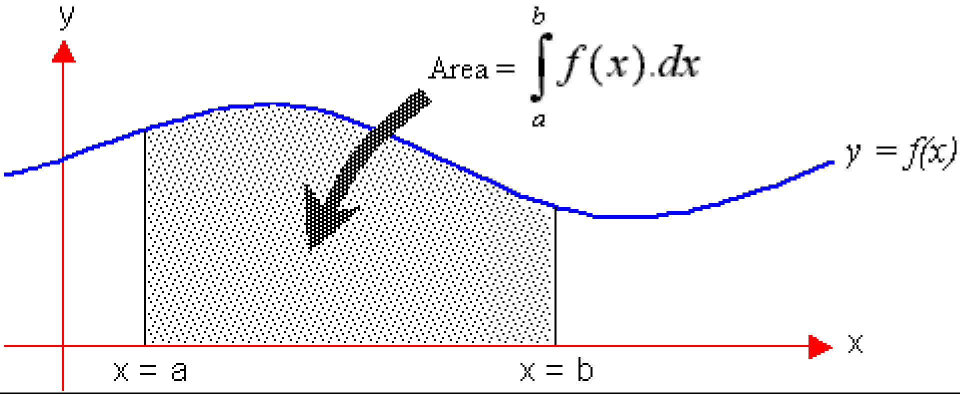
AP Readiness Area and Average Value

AREA UNDER A CURVE

By definition and The Fundamental Theorem of Calculus, the definite integral is the area bounded by a curve and the x axis in the interval [a,b]



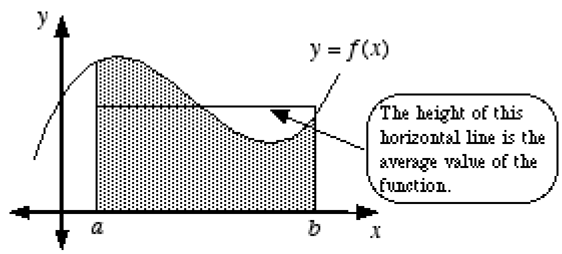
Find the area bounded by $f(x) = (x-4)^2$, the x-axis and the y-axis Desmos	Find the area bounded by f(x)= 2sin(x)+2 , the y-axis and x = $\frac{3\pi}{2}$ Desmos

Average Value

The Average Value of a Function in an interval [a,b] is the integral/net change divided by the length of the interval. A=b*h → average

Average value = $\frac{1}{b-a}\int_a^b f(x)\,dx$

The rectangle has the same area as the shaded region under the curve.



Find the average value of f(x) = $\frac{8}{x}$ on the interval [1,8] Desmos	Find the average value of f(x) = 2cos(x)+x on the interval $[0,\frac{5\pi}{6}]$ Desmos

Suppose the daily temperature variation can be modeled as a cosine function:

y = 74 + 6 cos[(10 + t)/4],

where *y* is temperature in degrees F and *t* is hours since midnight.

What is the average temperature throughout the day?

Desmos

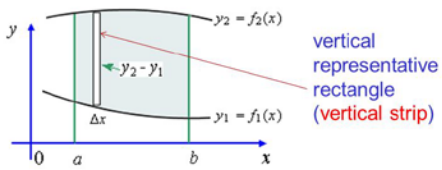
AP Readiness Area and Average Value
Area Between Two Curves

Vertical Strips

The formula for the area between curves is:

$$\text{Area} = \int_a^b \left[f_2(x) - f_1(x) \right] dx$$

$$Area = \int_a^b (top - bottom)\ dx$$



Horizontal Strips

Area= $\int_c^d [f_2(y) - g_2(y)\ dy$

Area= $\int_c^d [(right - left)dy$ [c is bottom, d is top]

Process

- To find the area between curves:
  1. Sketch the region defined in the problem.
  2. Connect the curves with either a vertical strip (dx) or a horizontal strip (dy).
     - A strip that always connects the two curves will allow you to find the area without breaking up integrals.
  3. Write an expression for the length of the rectangular strips.
     - **<u>Vertical Strips:</u>** Length = Top curve – Bottom curve
     - **<u>Horizontal Strips:</u>** Length = Right curve – Left curve
     - *<u>NOTE:</u> IF YOU USE A dy STRIP, YOU MUST SOLVE THE CURVE FOR x IN TERMS OF y.*
  4. Add rectangular strips together by setting up an integral using your expression.
  5. Find points of intersection.
     - *<u>NOTE:</u>* **If using a dx, use the x-coordinates of intersection. If using a dy, use the y-coordinates of intersection.**

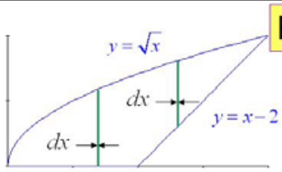
Find the area between f(x) = $-x^2+5$ and g(x)= x-1

[Desmos]

Find the area bounded by x=$-y^2$ +4 and x=y to 3 dp

[Desmos] * Requires Calculator

Example

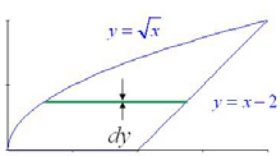


Find the area between these two curves from x =0 to x = 4.

If we try vertical strips, we have to integrate in two parts:

$$\int_0^2 \sqrt{x}\ dx + \int_2^4 \sqrt{x} - (x-2)\, dx$$

We can find the same area using a horizontal strip.



$y = \sqrt{x}$ $\quad y = x - 2$

$y^2 = x$ $\quad y + 2 = x$

Since the width of the strip is dy, we find the length of the strip by solving for x in terms of y.

[Desmos]

Vertical

Horizontal

Careful---- sometimes you need to integrate two separate areas when curves cross over each other and the tops and bottoms switch!

Find the area bounded by the y axis, x= $\frac{\pi}{2,}$, y = 2cos(x) and y = sin(x)

[Desmos]

- All Desmos links are on mrlangemath.com go to AP Readiness