AP Readiness

Extrema and Optimization

Key Ideas

Three Ways to Justify Extema

First Derivative Test

The First Derivative Test Suppose that c is a critical number of a continuous function f.

(a) If f' changes from positive to negative at c, then f has a local maximum at c.

(b) If f' changes from negative to positive at c, then f has a local minimum at c.

(c) If f' does not change sign at c (for example, if f' is positive on both sides of c or negative on both sides), then f has no local maximum or minimum at c.

What does this look like? Draw a picture

Second Derivative Test

Second Derivative Test

Let f be a function with $f'(c)=0$ and the **second derivative** exists on an open interval that contains ***c***:

1. If $f''(c)>0$, f has a relative **minimum** at ***x*** = ***c***. (Think "cup up")
2. If $f''(c)<0$, f has a relative **maximum** at ***x*** = ***c***. (Think "cup down")

Note that the second derivative test does not necessarily work when the first and **second derivatives are 0**, or **undefined**. It also doesn't say what happens at the endpoints of a function. In these situations, the **first derivative test** must be used.

What does this look like? Draw a picture.

Candidates Test [for absolute extrema on closed interal] aka The Closed Interval Test

I. Find the critical numbers (f'(x)=0 or does not exist). Make note of the **endpoints.** These are the only possible candidates to be extrema- hence the name "Candidates Test"..
II. Plug in the x values to find the y-values
III. The highest values are the absolute maxima and the lowest values are the absolute minima

Optimization

I. Look for "est" words- largest, smallest, greatest, furthest,most, least or words like maximum or minimum.
II. Draw and label a picture
III. Write equations relating variables to find a function you want to maximize/minimize in **one variable**.
IV. Take the derivative and find the maximum or minimum as needed
V. Check to make sure the answer makes sense and solves what is being asked.

AP Readiness

Extrema and Optimization

Complete each statement by choosing one of the four phrases from the box below. Phrases may be used more than once. Unless otherwise specified, assume each function is defined and continuous for all real numbers.

the absolute (global) maximum	the absolute (global) minimum
a local (relative) maximum	a local (relative) minimum

1. A function f defined and continuous on the interval $-2 \le x \le 5$ has critical points (or critical numbers) only at $x = -1$ and $x = 2$. The function f has values as given in the table below.

x	$f(x)$
-2	1
-1	2
2	0
5	2

The value $x = 2$ locates ___________________ value of the function. The value $f(x) = 2$ is ____________________ value of the function.

2. If $x = 2$ is the only critical point of a function f and $f''(2) < 0$, then $x = 2$ locates __________________ value of the function.

3. If $f'(2) = 0$ and $f'(x)$ changes from negative to positive at $x = 2$, then $x = 2$ locates

____________________ value of the function f.

4. If $f'(2) = 0$ and $f''(2) > 0$, then $x = 2$ locates ______________________ value of the function f.

5. If $x = 2$ is a critical point of the function f, and $f'(x)$ decreases through $x = 2$, then $x = 2$ locates __________________________ value of the function.

6. If a continuous function f increases throughout a closed interval, then the left endpoint of the graph of f on the interval is __________________________ point of the function.

7. A student found the critical points of a function f to be $x = 2$ and $x = 4$, and

produced the chart below.

Interval	$x < 2$	$2 < x < 4$	$x > 4$
$f'(x)$	positive	negative	positive

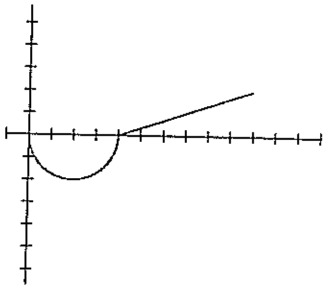
The value $x = 2$ locates _____________________________ value of the function.

8. If $x = 2$ is the only critical point of a function f and $f''(2) = 3$, then $x = 2$ locates a ______________________ value of the function.

Example 1 (noncalculator): Given the function $f\ x) = -\ x - e^{1-x}$ on the closed interval 0 3], convince me in three different ways that the maximum value of $f\ x)$ occurs at $x = 1$.

is the ONLY critical point.

Example 2 (graphical, noncalculator): The graph of $f'(x)$, shown below on the interval 0 10], consists of a semicircle of radius 2 and a line segment meeting at the point (4 0). The line segment has right endpoint (10 2).



Graph of f'

a. For what value of x, $0 < x < 10$, does f have a local minimum? Justify your answer.

b. For what value of x, $0 \leq x \leq 10$, does f have an absolute maximum? Justify your answer.

Example 3 (accumulating rates of change, calculator active): The rate, in gallons per hour, at which water flows into a tank over the time interval $0 \leq t \leq 3$, where t is measured in hours, is given by $R(t) = t^2 \sin(t)$ and the rate, in gallons per hour, at which water flows out of the tank over the same time interval is given by $S(t) = \frac{2}{(t+1)^2}$. The tank initially holds 10 gallons of water.

a. When is the rate at which water flows into the tank the greatest? Justify your answer.

b. Write an expression that represents the amount $A(t)$ of water in the tank at any given time t.

c. Determine when there is the least amount of water in the tank. How much water is in the tank at this time? Show the analysis that leads to your conclusion.

Example 4 (motion, noncalculator): The velocity of a particle moving on the x-axis is given by $v(t) = t^3 - 6t^2$ for the time interval $0 \leq t \leq 10$.

a. When is the particle farthest to the left?

b. When is the velocity of the particle increasing the fastest?