

Curve Sketching and Function Analysis Using Derivatives

Vocabulary: First Derivative, Second Derivative, Rate of Change, Increasing, Decreasing, Constant, Extrema, Absolute Maximum, Absolute Minimum, Relative/Local Maximum, Relative/Local Minimum, Concavity, Concave Up, Concave Down, Point of Inflection, Monotonic, x-intercept, y-intercept, Zeroes, Critical Points, End Behavior, Differentiable, Speed, Velocity, Acceleration, First Derivative Test, Second Derivative Test

What does the function look like when...?

| | $f'(x) > 0$ | $f'(x) < 0$ | $f'(x) = 0$ |
|--------------|-------------|-------------|-------------|
| $f''(x) > 0$ | | | |
| $f''(x) < 0$ | | | |
| $f''(x) = 0$ | | | |

What if the derivative does not exist at a point? What could that look like?

The First Derivative Test Suppose that c is a critical number of a continuous function f .

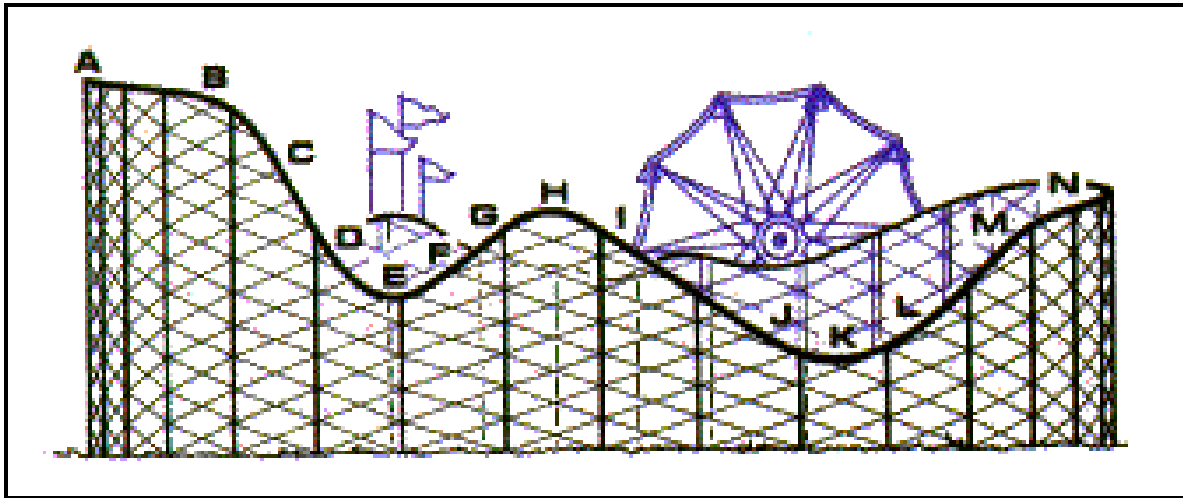
- If f' changes from positive to negative at c , then f has a local maximum at c .
- If f' changes from negative to positive at c , then f has a local minimum at c .
- If f' does not change sign at c (for example, if f' is positive on both sides of c or negative on both sides), then f has no local maximum or minimum at c .

The Second Derivative Test Suppose f'' is continuous near c .

- If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c .
- If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c .

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Describe the Coaster's Velocity and Acceleration at each point on the ride.



Let's Analyze Our Basic Functions (work them in your notebook)

What are the domain and range of the function? What are the x and y intercepts of the function? Over what intervals are the functions increasing/decreasing/constant? What are the critical values? Identify any extrema? Over what intervals are the function concave up/down? What are the points of inflection? What is the end behavior of the function?

- | | | |
|-----------------|-------------------------|-------------------------|
| 1. $f(x) = c$ | 6. $f(x) = e^x$ | 10. $f(x) = \sin(x)$ |
| 2. $f(x) = x$ | 7. $f(x) = \ln(x)$ | 11. $f(x) = \tan(x)$ |
| 3. $f(x) = x $ | 8. $f(x) = \sqrt{x}$ | 12. $f(x) = \arcsin(x)$ |
| 4. $f(x) = x^2$ | 9. $f(x) = \frac{1}{x}$ | |
| 5. $f(x) = x^3$ | | |

Let's sketch some more advanced functions using calculus

Use a Curve Sketching Template to Guide Your Way <http://www.mrlangemath.com/curvesketch.pdf>

| | |
|--------------------------------------|---|
| $f(x) = x^2 - 6x + 8$ | $f(x) = x^3 - 6x^2 + 9x + 1$ |
| $f(x) = x^{\frac{2}{3}}$ | $f(x) = \frac{2(x^2 - 9)}{x^2 - 4}$ |
| $f(x) = \frac{x}{x^2 - 1}$ | $f(x) = \sin(x) + \cos(x)$ on $[0, 2\pi]$ |
| $f(x) = \frac{6}{x^2} - \frac{6}{x}$ | $f(x) = -2x^4 + 4x^3$ |

AP Readiness

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