Integral as Area and Riemann Sums

AP Readiness

We have learned about **limits**. We have learned that the **derivative** measures the **slope** of the curve at any point by using limits. The limit definition of the derivative is $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

The definite **integral** represents the area under a curve. It can also be solved using limits.

We begin with Riemann Sums to approximate area under a curve.

$$S = \sum_{i=1}^{n} f(x_i^*)(x_i - x_{i-1}) \qquad x_{i-1} \le x_i^* \le x_i$$

Left Riemann sum: $x_i^* = x_{i-1}$ for all i

Right Riemann sum: $x_i^* = x_i$ for all i

Middle Riemann sum: $x_i^* = \frac{1}{2}(x_i + x_{i-1})$ for all i

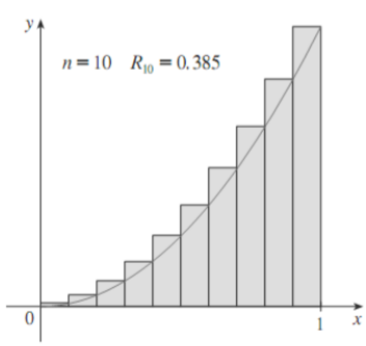
Approximate the area under the curve f(x)= x^2 on the interval [0,4] using:

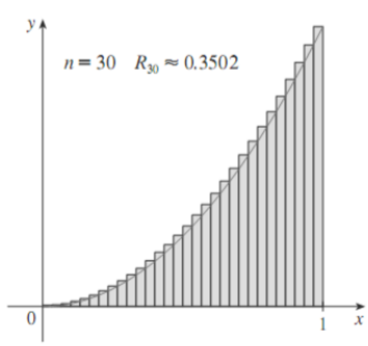
1. Left Rectangles with four subintervals: LRAM=
2. Right Rectangles with four subintervals RRAM=
3. Midpoint Rectangles with four subintervals MRAM=
4. Trapezoids with four subintervals TRAP=

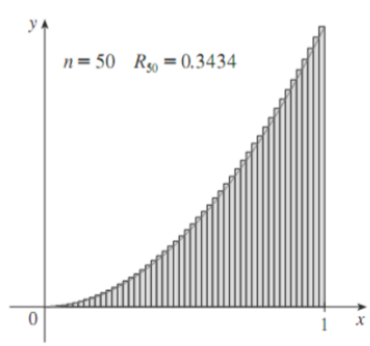
Limit Definition of the Integral

If we desire better approximations of the area, we could partition our area into smaller subintervals using more rectangles. The following chart shows the areas of the same region *S*, using *n* rectangles of equal width using both the left-endpoint and right-endpoint methods.

n	L_n	R_n
10	0.2850000	0.3850000
20	0.3087500	0.3587500
30	0.3168519	0.3501852
50	0.3234000	0.3434000
100	0.3282500	0.3383500
1000	0.3328335	0.3338335







$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

The Fundamental Theorem of Calculus: Part I

Suppose that *f* is bounded on the interval [*a*,*b*], and that *F* is an antiderivative of *f*, i.e., *F'* = *f*.

Then $\int_a^b f(x)dx = F(b) - F(a)$

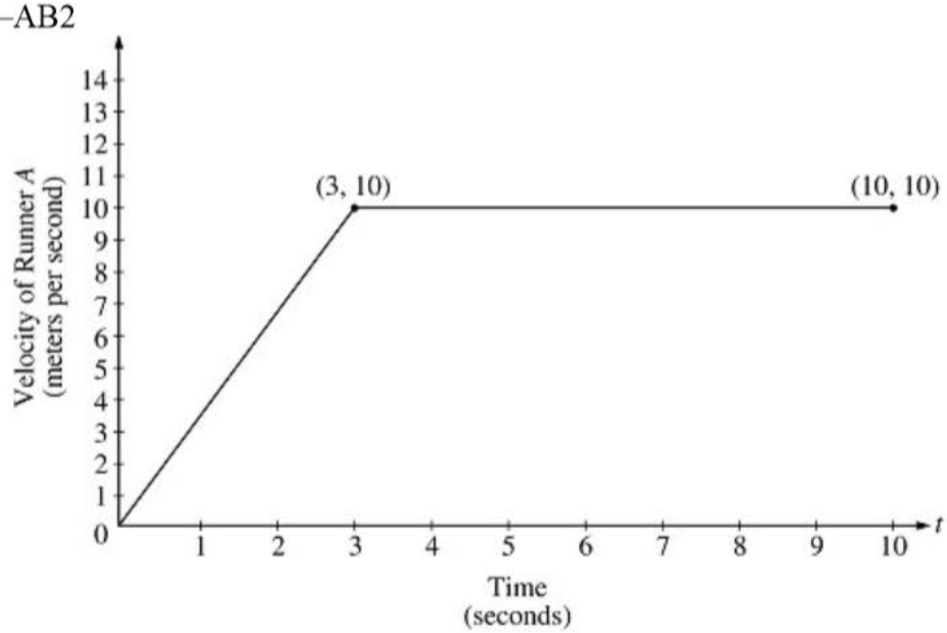
Find the exact area under $f(x)=x^2$ on [0,4] the curve using the FTC.

Verify the result using your calculator's ability to integrate. On a TI it is "Math 9".

11. If $f(x)$ is represented by the table below, approximate $\int_1^{9.6} f(x)dx$ using left-endpoint, right-endpoint, midpoint, and trapezoidal approximations. Label each one. Use as many subintervals as the data allows.

x	1	2.5	4	6	8	8.8	9.6	10.4
$f(x)$	4	3	1	3	5	6	4	7

12. 2000—AB2



Two runners, *A* and *B*, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner *A*. The velocity, in meters per second, of Runner *B* is given by the function *v* defined by $v(t) = \frac{24t}{2t+3}$.

(a) Find the velocity of Runner *A* and the velocity of Runner *B* at time *t* = 2 seconds. Indicate units of measure.

(b) Find the acceleration of Runner *A* and the acceleration of Runner *B* at time *t* = 2 seconds. Indicate units of measure.

(c) Find the total distance run by Runner *A* and the total distance run by Runner *B* over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.