Q. Fundamental Theorem of Calculus (FTC) / Accumulation Function

What you are finding: Students should certainly know that the FTC says that $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F(x)$ is an antiderivative of $f(x)$. This leads to the fact that $F(a) + \int_a^b f(x)\,dx$. Students should be prepared to use substitution methods to integrate and be able to change the limits of integration using this substitution. Students should also know that $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$.

The accumulation function looks like this: $F(x) = \int_0^x f(t)\,dt$. It represents the accumulated area under the curve f starting at zero (or some value) and going out to the value of x. The variable t is a dummy variable. It is important to believe that this is a function of x.

92. $\int_0^{\pi} e^{\cos x} \sin x \, dx =$

A. $\frac{1}{e}$ B. $e-\frac{1}{e}$ C. $e-1$ D. $1-\frac{1}{e}$ E. e

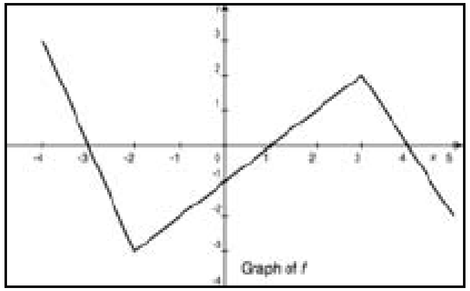
93. $\int_1^3 \sqrt{20-4x}\,dx$ is equivalent to

A. $\frac{-1}{4}\int_{-1/5}^{1/5} \sqrt{u}\,du$ B. $\frac{-1}{4}\int_{1}^{3} \sqrt{u}\,du$ C. $\frac{1}{4}\int_{8}^{16} \sqrt{u}\,du$ D. $4\int_{8}^{16} \sqrt{u}\,du$ E. $\int_{8}^{16} \sqrt{u}\,du$

94. $\int_1^x \frac{\ln t}{t}\,dt =$

A. $\frac{x^2-1}{2}$ B. $\frac{x^2}{2}$ C. $\frac{(\ln x)^2}{2}$ D. $\frac{(\ln x)^2-1}{2}$ E. $\ln x$

95. The graph of the piecewise linear function f is shown in the figure to right. If $g(x)=\int_{1}^{x} f(t)\,dt$, which of the following is the greatest?



A. $g(-4)$
B. $g(-3)$
C. $g(1)$
D. $g(4)$
E. $g(5)$

96. **(Calc)** Let $F(x)$ be an antiderivative of $\sqrt{x^3+x+1}$. If $F(1)=-2.125$, then $F(4)=$

A. -15.879 B. -11.629 C. 7.274 D. 15.879 E. 11.629

97. If f is a continuous function and $F'(x) = f(x)$ for all real numbers x, then $\int_{-2}^{2} f(1-3x)\,dx =$

A. $3F(-2) - 3F(2)$

B. $\frac{1}{3}F(-2) - \frac{1}{3}3F(2)$

C. $\frac{1}{3}F(2) - \frac{1}{3}3F(-2)$

D. $3F(-5) - 3F(7)$

E. $\frac{1}{3}F(7) - \frac{1}{3}F(-5)$

98. If f is continuous for all real numbers x and $\int_1^4 f(x)\,dx = 10$, then $\int_3^6 \left[f(x-2) + 2x \right] dx =$

A. 37 B. 39 C. 35 D. 57 E. 25

R. Derivative of Accumulation Function (2nd FTC)

What you are finding: You are looking at problems in the form of $\frac{d}{dx}\int_a^x f(t)\,dt$. This is asking for the rate of change with respect to x of the accumulation function starting at some constant (which is irrelevant) and ending at that variable x. It is important to understand that this expression is a function of x, not the variable t. In fact, the variable t in this expression could be any variable (except x).

How to find it: You are using the 2nd Fundamental Theorem of Calculus that says: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

Occasionally you may have to use the chain rule that says $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)$.

99. Let f be the function given by $f(x)=\int_x^{\pi/2} t\sin 2t\,dt$ for $-\pi\le x\le\pi$. In what interval(s) is $f(x)$ decreasing?

A. $(0,\pi)$

B. $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

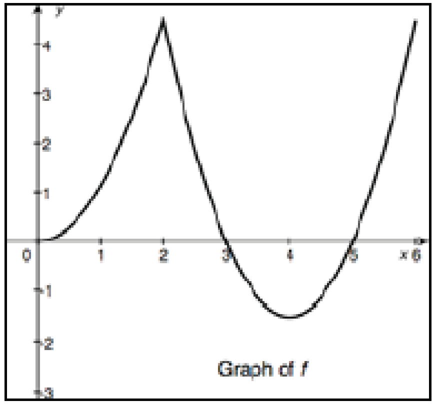
C. $\left(-\pi,\frac{-\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right)$

D. $(-\pi,0)$

E. $(-\pi,\pi)$

100. The graph of the function f shown to the right has a horizontal tangent at $x = 4$. Let g be the continuous function defined by $g(x) = \int_0^x f(t)\,dt$. For what value(s) of x does the graph of g have a point of inflection?

A. 3 and 5
B. 0, 3, and 5
C. 2 only
D. 2 and 4
E. 4 only



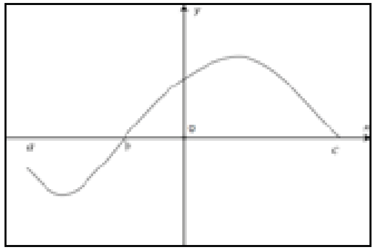
101. $\frac{d}{dx}\int_{1}^{x^3} \tan\left(t^4 - 1\right)\, dt =$

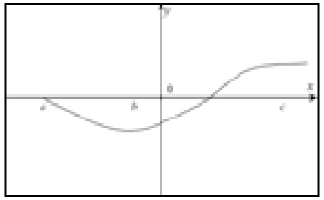
A. $\sec^2\left(x^{12} - 1\right)$
B. $\tan\left(x^4 - 1\right)$
C. $\tan\left(x^{12} - 1\right)$
D. $3x^2 \tan\left(x^{12} - 1\right)$
E. $12x^{11} \tan\left(x^{12} - 1\right)$

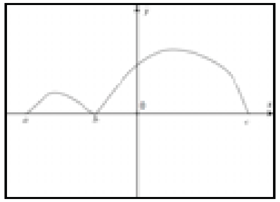
102. **(Calc)** Let f be the function given by $f(x)=\int_0^x \cos(t^2+t)\,dt$ for $-2\le x\le 2$. Approximately, for what percentage of values of x for $-2\le x\le 2$ is $f(x)$ decreasing?

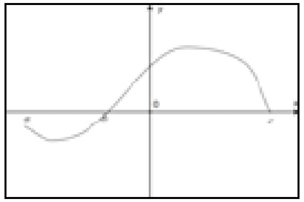
A. 30% B. 26% C. 44% D. 50% E. 59%

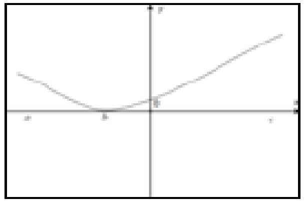
103. Let $f(x)=\int_a^x g(t)\,dt$ where g has the graph shown to the right. Which of the following could be the graph of f?

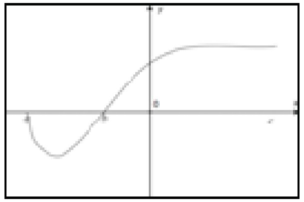


A. 

B. 

C. 

D. 

E. 

S. Interpretation of a Derivative as a Rate of Change

What you are finding: As mentioned in section I (Related Rates), a quantity that is given as a rate of change needs to be interpreted as a derivative of some function. Typical problems ask for the value of the function at a given time. These problems can be handled several ways:

a) solving a Differential Equation with initial condition (although DEQ's may not have even been formally mentioned yet)

b) Integral of the rate of change to give accumulated change. This uses the fact that:

$$\int_a^b R'(t)\,dt = R(b) - R(a) \text{ or } R(b) = R(a) + \int_a^b R'(t)\,dt$$

104. The table below gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval [-4, 4], what is the value of $\int_{-2}^{2} f'(2x)\,dx$?

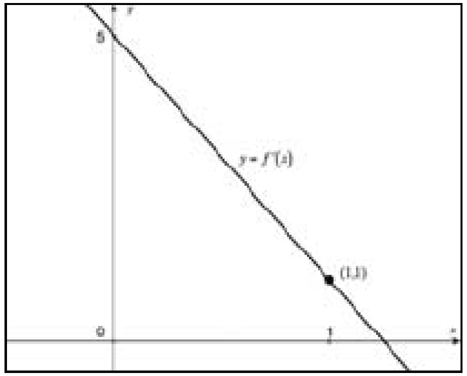
x	-4	-2	-1	1	2	4
$f(x)$	8	-6	-4	-2	-4	4
$f'(x)$	-6	-4	-2	0	2	4

A. -2 B. -8 C. 5 D. 20 E. 1

105. The graph of $y = f'(x)$, the derivative of f, is shown in the figure to the right. If $f(0) = -1$, then $f(1) =$

A. 1   B. 2   C. 3

D. 4   E. 5



106. **(Calc)** The rate of change of people waiting in line to buy tickets to a concert is given by $w(t) = 100(t^3 - 4t^2 - t + 7)$ for $0 \le t \le 4$. 800 people are already waiting in line when the box office opens at $t = 0$. Which of the following expressions give the change in people waiting in line when the line is getting shorter?

A. $\int_{1.480}^{3.773} w'(t)\, dt$

B. $\int_{1.480}^{3.773} w(t)\, dt$

C. $800 - \int_{1.480}^{3.773} w'(t)\, dt$

D. $\int_{0}^{2.786} w'(t)\, dt$

E. $\int_{0}^{2.786} w(t)\, dt$

107. **(Calc)** A cup of coffee is heated to boiling (212°F) and taken out of a microwave and placed in a 72°F room at time $t = 0$ minutes. The coffee cools at the rate of $16e^{-0.112t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the coffee at time $t = 5$ minutes?

A. 105°F B. 133°F C. 166°F D. 151°F E. 203°F

108. **(Calc)** The Cheesesteak Factory at a Philadelphia stadium has 25 steak sandwiches ready to sell when it opens for business. It cooks cheesesteaks at the rate of 4 steaks per minute and sells cheesesteaks at the rate of $1+6\sin\left(\frac{2\pi t}{41}\right)$ steaks per minute. For $0 \le t \le 20$, at what time is the number of steaks ready to sell at a minimum (nearest tenth of a minute)?

A. 0 B. 3.4 C. 10.3 D. 17.1 E. 20

T. Average Value of a Function

What you are finding: You are given a continuous function $f(x)$, either an algebraic formula or a graph, as well as an interval $[a,b]$. You wish to find the average value of the function on that interval.

How to find it: $f_{avg} = \dfrac{\int_a^b f(x)\,dx}{b-a}$. The units will be whatever the function *f* is measured in.

Again, be careful. The average rate of change of a function *F* on $[a,b]$ is not the same as the average value of the function *F* on $[a,b]$.

Average value of the function: $\dfrac{\int_a^b f(x)\,dx}{b-a}$

Average rate of change of the function (average value of the rate of change): $\dfrac{\int_a^b F'(x)\,dx}{b-a} = \dfrac{F(b)-F(a)}{b-a}$

109. The average value of $\sin^2 x \cos x$ on the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is

A. $\frac{-2}{3\pi}$ B. $\frac{2}{3\pi}$ C. 0 D. -1 E. 1

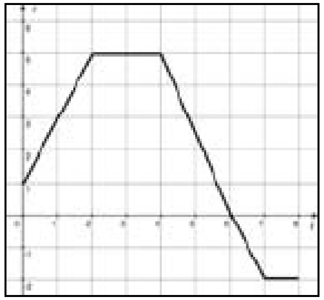
110. The velocity of a particle moving along the x-axis is given by the function $v(t) = te^{t^2}$. What is the average velocity of the particle from time $t = 1$ to $t = 3$?

A. $\dfrac{19e^9 - 3e}{2}$ B. $4e^9$ C. $\dfrac{3e^9 - e}{2}$ D. $\dfrac{e^9 - e}{4}$ E. $e^9 - e$

111. **(Calc)** What is the average value of $y = \dfrac{\sin(2x+3)}{x^2+2x+3}$ on the interval [-2, 3] ?

A. -0.057 B. 0.051 C. 0.061 D. 0.256 E. 0.303

112. Newton the cat begins to walk along a ledge at time $t = 0$. His velocity at time t, $0 \leq t \leq 8$, is given by the function whose graph is given in the figure to the right. What is Newton's average speed from $t = 0$ to $t = 8$?



A. 0

B. 2

C. 3

D. $\frac{9}{4}$

E. 5

113. **(Calc)** Barstow, California has the daily summer temperature modeled by $T(t) = 85 + 27\sin\left[\frac{\pi(x-9)}{12}\right]$ where t is measured in hours and $t = 0$ corresponds to 12:00 midnight. What is the average temperature in Barstow during the period from 1:30 PM to 6 PM?

A. 60° B. 89° C. 104° D. 110° E. 115°

114. Let $f(x) = 3x^2 - 6x$. For how many positive values of b does the average value of $f(x)$ on the interval [0, b] equal the average rate of change of $f(x)$ on the interval [0, b] ?

A. 4 B. 3 C. 2 D. 1 E. none