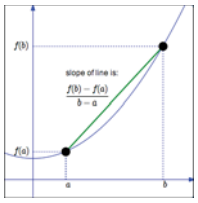
Average vs. Instantaneous Rates of Change: Discovering the Derivative
AP Readiness - Mr. Lange, Hollywood High School

Average Rate of Change is the slope of the secant line between two points.

$$A.R.C. = \frac{\text{change in } f(x)}{\text{change in } x} = \frac{f(x) - f(a)}{x - a}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{f(b) - f(a)}{b - a}$$

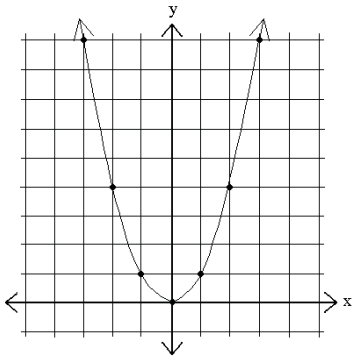


$$Average\ Velocity = \frac{Change\ in\ position\ (displacement)}{Change\ in\ time} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$Average\ Acceleration = \frac{Change\ in\ velocity}{Change\ in\ time} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

1. $f(x) = x^2$ Find the average rate of change on the following intervals:

[-2,2]	[0,2]
[1,2]	[2,3]
[1.5,2]	[1.9,2]
[1.99,2]	[2,2.5]
[2,2.1]	[2,2.01]
What do you notice as Δx gets smaller?	



x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

What is the rate of change of f(x) at x=2?

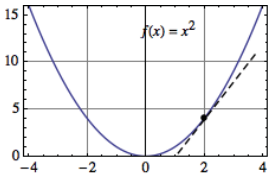
Instantaneous rate of change is the **slope of the tangent line** at a particular point, which happens to be the value of the derivative at that point. It is the limit of the average rate of change as Δx or h approaches zero or as x approaches a, depending on your definition.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\text{Average rate of change} = \frac{f(a+h) - f(a)}{h}$$

$$\text{Instantaneous rate of change} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



Let's verify the instantaneous rate of change (the derivative) of $f(x) = x^2$ at x=2

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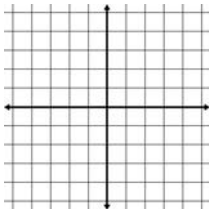
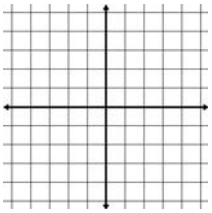
Let's look at some of the graphs in our family of functions in the context of rate of change and determine the derivatives.

Remember the limit definition of the derivative is $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

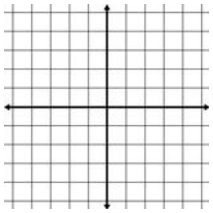
f(x)=c



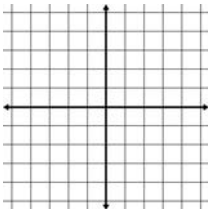
f(x)=x



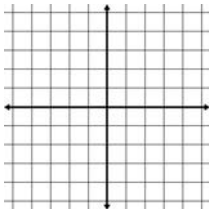
f(x)=x^2



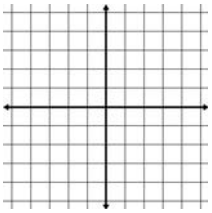
f(x)=x^3



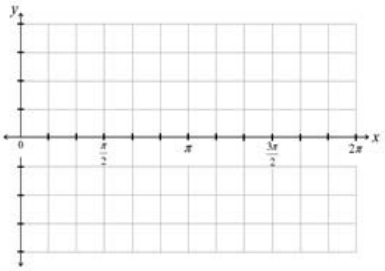
f(x)=$\sqrt{x}$



f(x)=|x|



f(x)= sin(x)



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