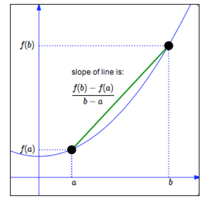
UCLA AP Readiness Sept 19- Mr. Lange, Hollywood High School
Average vs. Instantaneous Rate of Change: Discovering the Derivative

Average Rate of Change is the slope of the secant line between two points.

$$A.R.C. = \frac{\text{change in } f(x)}{\text{change in } x} = \frac{f(x) - f(a)}{x - a}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{f(b) - f(a)}{b - a}$$

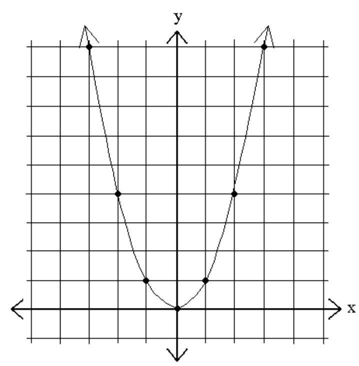


$$Average\ Velocity = \frac{Change\ in\ position\ (displacement)}{Change\ in\ time} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$Average\ Acceleration = \frac{Change\ in\ velocity}{Change\ in\ time} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

1. $f(x)= x^2$ Find the average rate of change on the following intervals:

[-2,2]	[0,2]
[1,2]	[2,3]
[1.5,2]	[1.9,2]
[1.99,2]	[2,2.5]
[2,2.1]	[2,2.01]
What do you notice as Δx gets smaller?	



x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

What is the rate of change of f(x) at x=2?

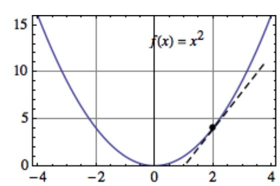
Instantaneous rate of change is the **slope of the tangent line** at a particular point, which happens to be the value of the derivative at that point. It is the limit of the average rate of change as Δx or h approaches zero.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\text{Average rate of change} = \frac{f(a+h) - f(a)}{h}$$

$$\text{Instantaneous rate of change} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



Let's verify the instantaneous rate of change (the derivative) of $f(x)= x^2$ at x=2

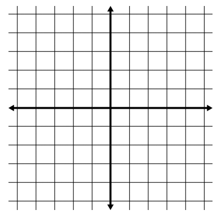
UCLA AP Readiness Sept 19- Mr. Lange, Hollywood High School

Average vs. Instantaneous Rate of Change: Discovering the Derivative

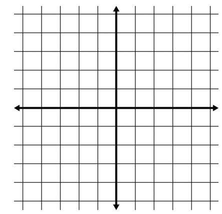
Let's look at some of the graphs in our family of functions in the context of rate of change and determine the derivatives.

Remember the limit definition of the derivative is $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

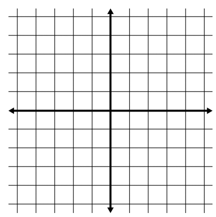
f(x)=c



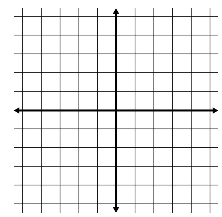
f(x)=x



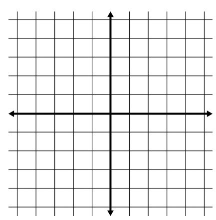
f(x)=x^2



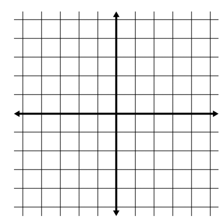
f(x)=x^3



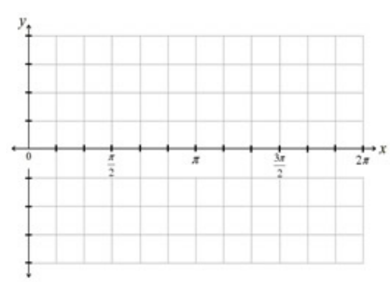
f(x)=$\sqrt{x}$



f(x)=|x|



f(x)= sin(x)



Great Calculus Resources			
Google	Desmos.com	mrlangemath.com	wolframalpha.com
Ms. Roshan's Screencasts- for Stewart	"Geogebra Calculus Applets"	"Calculus Maximus"	mastermathmentor.com
Find a graphing calculator for your phone	Youtube: PatrickJMT	KhanAcademy.org	Download Calculus Apps for your phone/tablets
Wikipedia			

LIMITS(1)

LIMIT LAWS

1. $\lim_{x\to 0}\frac{\sin x}{x}=1$ 2. $\lim_{x\to\infty}\frac{\sin x}{x}=0$ 3. $\lim_{x\to 0}\frac{\sin^2 x}{x}=0$

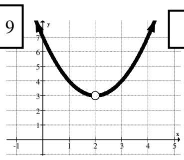
4. $\lim_{x\to 0}\frac{\cos x-1}{x}=0$ 5. $\lim_{x\to 0}\frac{\sin(ax)}{(bx)}=\lim_{x\to 0}\frac{\sin(ax)}{\sin(bx)}=\lim_{x\to 0}\frac{(bx)}{\sin(ax)}=\frac{a}{b}$

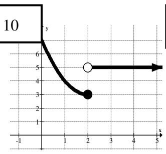
6. $\lim_{x\to a} f(x)=L$ (exists) If and only if $\lim_{x\to a^+} f(x)=\lim_{x\to a^-} f(x)=L$

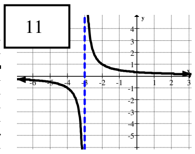
7. $f(x)$ is cont a if $\lim_{x\to a} f(x)=f(a)$

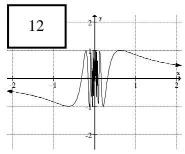
8. **Continuity at a if** $\lim_{x\to a^-} f(x)=f(a)=\lim_{x\to a^-} f(x)$

TYPES OF DISCONTINUITY

9

Point or removable

10

Jump

11

Infinite

12

Oscillating

ARITHMETIC OF INFINITY

1. $\infty+\infty=\infty$ 2. $n+\infty=\infty$ 3. $\infty+0=\infty$ 4. $0+\infty=\infty$ $\}(+)$	1. $\infty\bullet\infty=\infty$ 2. $n\bullet\infty=\infty$ 3. $0\bullet\infty=0$ $\}(\times)$
1. $\infty-\infty=und$ 2. $n-\infty=-\infty$ 3. $\infty-n=\infty$ 4. $n-n^-=0^-=-\frac{1}{\infty}$ 5. $n^+-n=0^+=\frac{1}{\infty}$ $\}(-)$	1. $\infty/\infty=und$ 2. $n/\pm\infty=0$ 3. $\infty/n=\infty$ 4. $n/0=\pm\infty$ 5. $n/0^+=\infty$ 6. $n/0^-=-\infty$ $\}(\div)$
1. $\infty^\infty=\infty$ 2. $\infty^n=\infty$ 3. $\infty^0=1$ 5. $0^\infty=0$ 4. $n^\infty=\infty$ $\}$ power	1. $-1\le\sin(\pm\infty)\le 1$ 2. $-1\le\cos(\pm\infty)\le 1$ $\}$ Trig

DERITAVITIVES(2)

Basic Rules

1. $\frac{d}{dx}c=0$ (Constant) 2. $\frac{d}{dx}c[f(x)]=c\frac{d}{dx}f(x)$ (constant multiple)

3. $\frac{d}{dx}x^n=nx^{n-1}$ (power) 4. $\frac{d}{dx}(u\pm v)=\frac{d}{dx}u\pm\frac{d}{dx}v$ (Sum & Difference)

5. $\frac{d}{dx}(uv)=v\frac{d}{dx}u+u\frac{d}{dx}v$ (Product) 6. $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{d}{dx}u-u\frac{d}{dx}v}{v^2}$ (Quotient)

7. $\left(f^{-1}\right)'(x)=\frac{1}{f'\left(f^{-1}(x)\right)}$ (Inverse)

8. $\frac{d}{dx}f(g(x))=\frac{d}{dx}f(u)\bullet\frac{d}{dx}g(x)$ where $u=g(x)$ (Quotient)

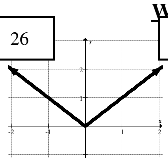
Exponential and Logarithmic Functions

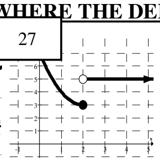
20. $\frac{d}{dx}\ln u=\frac{1}{u}\frac{du}{dx}$ 21. $\frac{d}{dx}\log_a u=\frac{1}{u\ln a}\frac{du}{dx}$

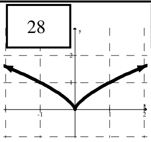
22. $\frac{d}{dx}e^u=e^u\frac{du}{dx}$ 23. $\frac{d}{dx}a^u=a^u\ln a\frac{du}{dx}$

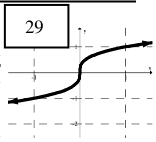
24. $\frac{d}{dx}\sqrt{u}=\frac{1}{2\sqrt{u}}\frac{du}{dx}$ 25. $\frac{d}{dx}|u|=\frac{u}{|u|}\frac{du}{dx}$

WHERE THE DERIVATIVE DOES NOT EXIST

26

Corner

27

Discontinuity

28

Cusp

29

Vertical tangent

WWW.MATHGOTSERVED.COM

Linearization: $L(x)=f(a)+f'(a)(x-a)$

Trigonometric Functions

8. $\frac{d}{dx}\sin u=\cos u\frac{du}{dx}$ 9. $\frac{d}{dx}\cos u=-\sin u\frac{du}{dx}$

10. $\frac{d}{dx}\tan u=\sec^2 u\frac{du}{dx}$ 11. $\frac{d}{dx}\cot u=-\csc^2 u\frac{du}{dx}$

12. $\frac{d}{dx}\sec u=\sec u\tan u\frac{du}{dx}$ 13. $\frac{d}{dx}\csc u=-\csc u\cot u\frac{du}{dx}$

Inverse Trigonometric Functions

14. $\frac{d}{dx}\sin^{-1}u=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ 15. $\frac{d}{dx}\cos^{-1}u=\frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$

16. $\frac{d}{dx}\tan^{-1}u=\frac{1}{1+u^2}\frac{du}{dx}$ 17. $\frac{d}{dx}\cot^{-1}u=\frac{-1}{1+u^2}\frac{du}{dx}$

18. $\frac{d}{dx}\sec^{-1}u=\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$ 19. $\frac{d}{dx}\csc^{-1}u=\frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

30. Mean Value Theorem: If f is cont on $[a,b]$ on and diff on (a,b) $\Rightarrow$ exist a $c\in(a,b)$ s.t. $f'(c)=\frac{f(b)-f(a)}{b-a}$

31. Rolle's Theorem: MVT where $f'(c)=\frac{f(b)-f(a)}{b-a}=0$

32. IntermediateValueTheorem.: If f is cont on $[a,b]$ and $d\in[f(a),f(b)]$ then there is a $c\in[a,b]$ st $f(c)=d$

Definition of Derivative

33. $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

34. $f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

35. $f'(x)\approx$ **Average rate of change = Slope of Secant line** $=\frac{f(b)-f(a)}{b-a}$

INTEGRALS (3)

APPROXIMATING AREA

11. $LRAM_n=w\left(f(x_1)+f(x_2)+..+f(x_{n-1})\right)$ **or** $w_1f(x_1)+w_2f(x_2)+..+w_{n-1}f(x_{n-1})$

12. $RRAM_n=w\left(f(x_2)+f(x_3)+\cdots+f(x_n)\right)$ **or** $w_1f(x_2)+w_2f(x_3)+..+w_{n-1}f(x_n)$

13. $MRAM_n=w\left(f\left(\frac{x_1+x_2}{2}\right)+f\left(\frac{x_2+x_3}{2}\right)+\cdots+f\left(\frac{x_{n-1}+x_n}{2}\right)\right)$ *or* $w_1f\left(\frac{x_1+x_2}{2}\right)+w_2f\left(\frac{x_2+x_3}{2}\right)+\cdots+w_{n-1}f\left(\frac{x_{n-1}+x_n}{2}\right)$

Note: $w=\frac{b-a}{n}$ and applies only for equal sub intervals

14. $T_n=\frac{w}{2}\left(y_1+2y_2+\cdots+2y_{n-1}+y_n\right)$ **or** $\frac{1}{2}\left(w_1(y_1+y_2)+w_2(y_2+y_3)+...\right)$

20. Area $=\int_{x_{left}}^{x_{right}}\left[f(x)_{top}-f(x)_{down}\right]dx$ or Area $=\int_{y_{down}}^{y_{top}}\left[f(y)_{left}-f(y)_{right}\right]dy$

21. Vol of rev $=\pi\int_{x_{left}}^{x_{right}}\left\{\left[f(x)_{top}-a\right]^2-\left[f(x)_{down}-a\right]^2\right\}dx$ Vol Cross sect.$=\int_a^b A(x)dx$

ANTIDIFFERENTION (INTEGRATION) RULES

1. $\int x^n dx=\frac{x^{n+1}}{n+1}+C$ 2. $\int\frac{1}{x}dx=\ln|x|+C$ 3. $\int\frac{1}{ax+b}dx=\frac{\ln|ax+b|}{a}+C$ 4. $\int e^{kx}dx=\frac{e^{kx}}{k}+C$

5. $\int a^x dx=\frac{a^u}{\ln a}+C$ 6. $\int\sin kx\,dx=-\frac{\cos kx}{k}+C$ 7. $\int\cos kx\,dx=\frac{\sin kx}{k}+C$

8. $\int\sec x\tan x\,dx=\sec x+C$ 9. $\int\sec^2 x\,dx=\tan x+C$ 10. $\int\csc x\cot x\,dx=-\csc x+C$

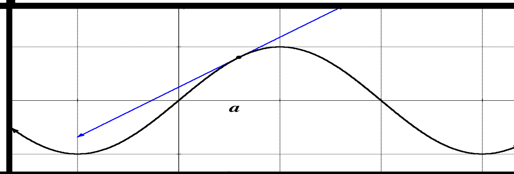
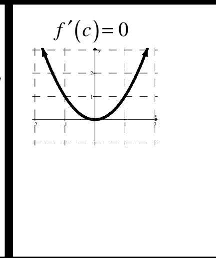
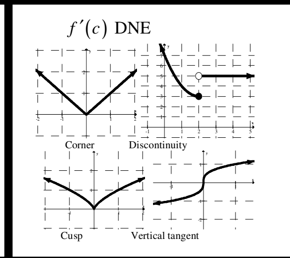
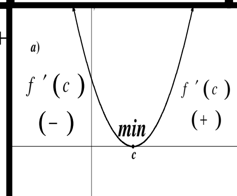
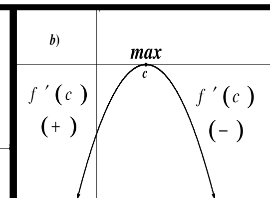
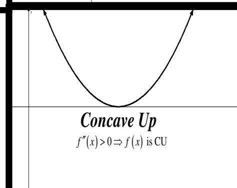
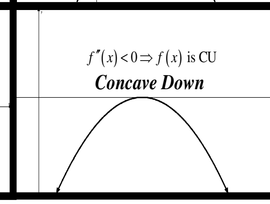
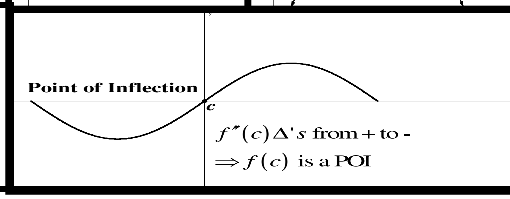
15. **Average Value of f** $\text{av}(f)=f_{ave}=\frac{1}{b-a}\int_a^b f(x)dx$

16. FTC I: $\int_a^b f'(x)dx=f(b)-f(a)$ 17 FTC II i) $\int_a^x f(t)dt=F(x)$

ii) $\frac{d}{dx}\int_a^x f(t)dt=f(x)$ iii) $\frac{d}{dx}\int_{h(x)}^{g(x)}f(t)dt=f(g(x))\cdot g'(x)-h(g(x))\cdot h'(x)$

18. Integration by parts: $\int v\,du=uv-\int v\,du$ use LIPET to select u

19. Integration by substitution: $\int f(g(x))g'(x)=\int f(u)du$

Term	Verbal Description	Symbolic	Graphical
1. Derivative of f at a:	The instantaneous rate of change of the function at a or the slope of the tangent line at a	$f'(a) = \left.\frac{df}{dx}\right\|_{x=a}$ $= \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$	
2. Critical Number c	A number c in an open (a, b) interval where the derivative is zero or does not exist	$c \in (a,b)$ where $f'(c)=0$ or $f'(c)$ DNE	 
3. First Derivative Test	a) If the first derivative changes from negative to positive at c then the function has a relative minimum at c b) If the first derivative changes from positive to negative at c then the function has a relative maximum at c	a) If $f'(c)$ Δ's from $-$ to $+$ $\Rightarrow f'(c)$ is a min b) If $f'(c)$ Δ's from $+$ to $-$ $\Rightarrow f'(c)$ is a max	 
4. Concavity Test	a) If the second derivative is positive on an interval I then the function is Concave Up on I b) If the second derivative is negative on an interval I the function is Councave down on I	a) If $f''(c)>0$ on I $\Rightarrow f(x)$ is CU on I b) If $f''(c)<0$ on I $\Rightarrow f(x)$ is CD on I	 
5. Point of Inflection at c	**f: Is a point where the concavity of f changes** **f': Is a point where f' changes from increasing to decreasing or decreasing to increasing** **f'': Is a point where f'' changes from positive to negative or negative to positive**	f Δ's from CU to CD or CD to CU f' Δ's from ↗ to ↘ or ↘ to ↗ $f''(x)$ Δ's from + to − or − to	

VOCABULARY (4)

<u>Motion definitions and Equations</u>

6. Displacement: A Vector quantity that represents the net change in position	$s(t) = x(b) - x(a) = \int_a^b v(t)$	7. Distance: A scalar quantity that represents total movement regardless of sign	$d(t) = \|x(b)-x(a)\| = \int_a^b \|v(t)\|dt$
8. Velocity: A Vector quantity that represents the rate of change of position	$v(t) = s'(t)$	9. Speed: A scalar quantity that represents the rate of covering distance	Speed $= \|v(t)\|$
10. Acceleration: A vector quantity that represents the rate of change of velocity	$a(t) = v'(t) = s''(t)$	11. Given initial position $s(a) = C$ the final position is given by $s(b) = s(a) + \int_a^b s'(t)\,dt$	

<u>Reciprocal</u>

$\sin x = \frac{1}{\csc x}$ $\csc x = \frac{1}{\sin x}$

$\cos x = \frac{1}{\sec x}$ $\sec x = \frac{1}{\cos x}$

$\tan x = \frac{1}{\cot x}$ $\cot x = \frac{1}{\tan x}$

<u>Quotient</u>

$\tan x = \frac{\sin x}{\cos x}$

$\cot x = \frac{\cos x}{\sin x}$

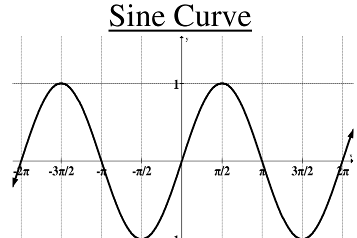
<u>Pythagorean</u>

$\sin^2 x + \cos^2 x = 1$

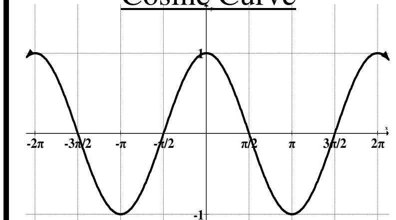
$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

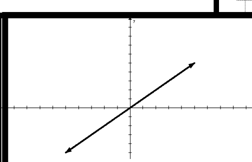
<u>Sine Curve</u>



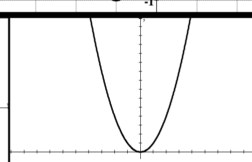
<u>Cosine Curve</u>



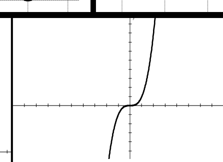
	0	$\pi/6$ (30°)	$\pi/4$ (45°)	$\pi/3$ (60°)	$\pi/2$ (90°)
$\sin x$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos x$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan x$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Und.
$\csc x$	Und.	2	$2/\sqrt{2}$	$2/\sqrt{3}$	1
$\sec x$	1	$2/\sqrt{3}$	$2/\sqrt{2}$	2	Und.
$\cot x$	Und.	$2/\sqrt{3}$	1	$1/\sqrt{3}$	0



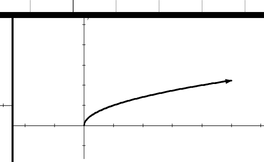
Linear $y = x$



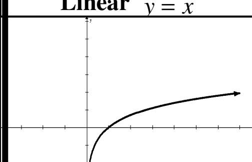
quadratic $y = x^2$



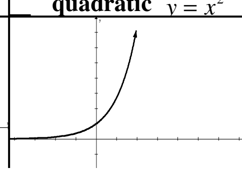
Cubic $y = x^3$



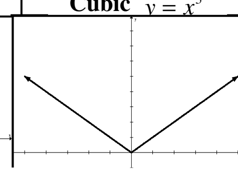
Radical $y = \sqrt{x}$



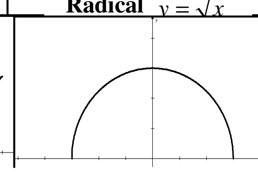
Logarithmic $y = \ln x$



Exponential $y = e^x$



Absolute value $y = |x|$



circular $y = \sqrt{9 - x^2}$