AP Calculus Test Information, Tips, and Common Errors

Exam Format:

Multiple Choice – 50% of grade
- Part A: 28 questions, no calculator, 55 minutes
- Part B: 17 questions, calculator, 50 minutes

Free Response – 50% of grade
- 2 questions, calculator, 30 minutes
- 4 questions, no calculator, 60 minutes

Tips

- *Show all work* – Remember that the grader is not really interested in finding out the answer to the problem. The grader is interested in seeing how you solved the problem.
- *Do not round intermediate answers* – Store them in your calculator (STO→) so that you can later use the exact answer.
- *Do not let points at the beginning keep you from getting points at the end* – If you can do part (c) without doing (a) or (b), do that. If you need to import an answer from part (a) to do part (c), make a credible attempt at part (a) so that you can import an answer (even if it is the wrong one) to finish part (c).
- *If you use your calculator to solve an equation/integral, write the equation/integral first* – An answer without an equation/integral may not get full credit, even if it is correct.
- *Do not waste time erasing bad solutions* – If you change your mind, simply cross out the bad solution. *Crossed-out work will not be graded*. If you have no better solution, leave the old solution because it might be worth a point or two.
- *Do not use your calculator for anything except:* (a) graphing functions, (b) computing numerical derivatives, (c) computing numerical integrals, and (d) solving equations. DO NOT use your calculator to determine min/max points, concavity, inflection points, increasing/decreasing intervals, domain, or range. (You can explore/verify all of these with your calculator, but your solution must be supported by calculus.)
- *Be sure you have answered the question (including units if they ask for it)* – For example, if it asks for the maximum values of a function, do not stop after finding the x-value (where it occurs). Be sure to express your answer in correct units if units are given.
- *If you can eliminate some incorrect answers in the multiple-choice section, it is to your advantage to guess* – Wrong answers can often be eliminated by estimation or graphing.
- *If they ask you to justify your answer, think about what needs justification* – They are asking you to say more. Write your answer in one or two short, clear, concise sentences. Do not ramble. Work is NOT justification (*including sign charts*).

Top Ten Student Mistakes

- If $f'(x)=0$, then there must be a max/min at that point! Not always true, use a sign chart.
- If $f''(x)=0$, then there must be an inflection point! Not always true, use a sign chart.
- Average rate of change of *f* on [*a*, *b*] is $\frac{f(b)-f(a)}{b-a}$, NOT $\frac{f'(a)+f'(b)}{2}$.
- Average value of a *f* on [*a*, *b*] is $\frac{1}{b-a}\int_a^b f(x)dx$, NOT $\frac{f(a)+f(b)}{2}$.
- Volume by washers is $\pi\int_a^b \left(R^2-r^2\right)dx$, NOT $\pi\int_a^b (R-r)^2\,dx$.
- Omitting the constant of integration.
- Assuming graders will know what “it” or the other pronouns refer to.
- If the correct answer came from your calculator, the grader will assume the setup was correct. You must show where your answer came from.
- $\int \frac{1}{x}dx=\ln|x|+C$, but $\int \frac{1}{f(x)}dx \neq \ln|f(x)|+C$
- Chain Rule errors…

LIMITS(1)

LIMIT LAWS

1. $\lim_{x\to 0}\frac{\sin x}{x}=1$ 2. $\lim_{x\to\infty}\frac{\sin x}{x}=0$ 3. $\lim_{x\to 0}\frac{\sin^2 x}{x}=0$

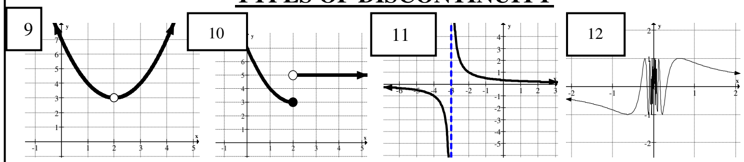
4. $\lim_{x\to 0}\frac{\cos x-1}{x}=0$ 5. $\lim_{x\to 0}\frac{\sin(ax)}{(bx)}=\lim_{x\to 0}\frac{\sin(ax)}{\sin(bx)}=\lim_{x\to 0}\frac{(bx)}{\sin(ax)}=\frac{a}{b}$

6. $\lim_{x\to a} f(x)=L$ (exists) If and only if $\lim_{x\to a^+} f(x)=\lim_{x\to a^-} f(x)=L$

7. $f(x)$ is cont a if $\lim_{x\to a} f(x)=f(a)$

8. **Continuity at a if** $\lim_{x\to a^-} f(x)=f(a)=\lim_{x\to a^-} f(x)$

TYPES OF DISCONTINUITY



9 Point or removable — 10 Jump — 11 Infinite — 12 Oscillating

ARITHMETIC OF INFINITY

1. $\infty+\infty=\infty$ 2. $n+\infty=\infty$ 3. $\infty+0=\infty$ 4. $0+\infty=\infty$ } (+)	1. $\infty\cdot\infty=\infty$ 2. $n\cdot\infty=\infty$ 3. $0\cdot\infty=0$ } (×)
1. $\infty-\infty=und$ 2. $n-\infty=-\infty$ 3. $\infty-n=\infty$ 4. $n-n^-=0^-=-\frac{1}{\infty}$ 5. $n^+-n=0^+=\frac{1}{\infty}$ } (−)	1. $\infty/\infty=und$ 2. $n/\pm\infty=0$ 3. $\infty/n=\infty$ 4. $n/0=\pm\infty$ 5. $n/0^+=\infty$ 6. $n/0^-=-\infty$ } (÷)
1. $\infty^\infty=\infty$ 2. $\infty^n=\infty$ 3. $\infty^0=1$ 5. $0^\infty=0$ 4. $n^\infty=\infty$ } power	1. $-1\le\sin(\pm\infty)\le 1$ 2. $-1\le\cos(\pm\infty)\le 1$ } Trig

DERITAVITIVES(2)

Basic Rules

1. $\frac{d}{dx}c=0$ (Constant) 2. $\frac{d}{dx}c[f(x)]=c\frac{d}{dx}f(x)$ (constant multiple)

3. $\frac{d}{dx}x^n=nx^{n-1}$ (power) 4. $\frac{d}{dx}(u\pm v)=\frac{d}{dx}u\pm\frac{d}{dx}v$ (Sum & Difference)

5. $\frac{d}{dx}(uv)=v\frac{d}{dx}u+u\frac{d}{dx}v$ (Product) 6. $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{d}{dx}u-u\frac{d}{dx}v}{v^2}$ (Quotient)

7. $(f^{-1})'(x)=\frac{1}{f'(f^{-1}(x))}$ (Inverse)

8. $\frac{d}{dx}f(g(x))=\frac{d}{dx}f(u)\cdot\frac{d}{dx}g(x)$ where $u=g(x)$ (Quotient)

Trigonometric Functions

8. $\frac{d}{dx}\sin u=\cos u\frac{du}{dx}$ 9. $\frac{d}{dx}\cos u=-\sin u\frac{du}{dx}$

10. $\frac{d}{dx}\tan u=\sec^2 u\frac{du}{dx}$ 11. $\frac{d}{dx}\cot u=-\csc^2 u\frac{du}{dx}$

12. $\frac{d}{dx}\sec u=\sec u\tan u\frac{du}{dx}$ 13. $\frac{d}{dx}\csc u=-\csc u\cot u\frac{du}{dx}$

Inverse Trigonometric Functions

14. $\frac{d}{dx}\sin^{-1}u=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ 15. $\frac{d}{dx}\cos^{-1}u=\frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$

16. $\frac{d}{dx}\tan^{-1}u=\frac{1}{1+u^2}\frac{du}{dx}$ 17. $\frac{d}{dx}\cot^{-1}u=\frac{-1}{1+u^2}\frac{du}{dx}$

18. $\frac{d}{dx}\sec^{-1}u=\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$ 19. $\frac{d}{dx}\csc^{-1}u=\frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

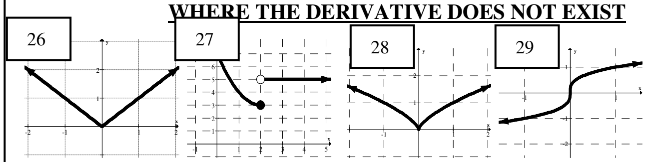
Exponential and Logarithmic Functions

20. $\frac{d}{dx}\ln u=\frac{1}{u}\frac{du}{dx}$ 21. $\frac{d}{dx}\log_a u=\frac{1}{u\ln a}\frac{du}{dx}$

22. $\frac{d}{dx}e^u=e^u\frac{du}{dx}$ 23. $\frac{d}{dx}a^u=a^u\ln a\frac{du}{dx}$

24. $\frac{d}{dx}\sqrt{u}=\frac{1}{2\sqrt{u}}\frac{du}{dx}$ 25. $\frac{d}{dx}|u|=\frac{u}{|u|}\frac{du}{dx}$

WHERE THE DERIVATIVE DOES NOT EXIST



26 Corner — 27 Discontinuity — 28 Cusp — 29 Vertical tangent

WWW.MATHGOTSERVED.COM

30. Mean Value Theorem: If f is cont on $[a,b]$ on and diff on (a,b) $\Rightarrow$ exist a $c\in(a,b)$ s.t. $f'(c)=\frac{f(b)-f(a)}{b-a}$

31. Rolle's Theorem: MVT where $f'(c)=\frac{f(b)-f(a)}{b-a}=0$

32. IntermediateValueTheorem.: If f is cont on $[a,b]$ and $d\in[f(a),f(b)]$ then there is a $c\in[a,b]$ st $f(c)=d$

Definition of Derivative

33. $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

34. $f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

35. $f'(x)\approx$ **Average rate of change = Slope of Secant line** $=\frac{f(b)-f(a)}{b-a}$

Linearization: $L(x)=f(a)+f'(a)(x-a)$

INTEGRALS (3)

APPROXIMATING AREA

11. $LRAM_n=w(f(x_1)+f(x_2)+..+f(x_{n-1}))$ **or** $w_1f(x_1)+w_2f(x_2)+..+w_{n-1}f(x_{n-1})$

12. $RRAM_n=w(f(x_2)+f(x_3)+\cdots+f(x_n))$ **or** $w_1f(x_2)+w_2f(x_2)+..+w_{n-1}f(x_n)$

13. $MRAM_n=w\left(f\left(\frac{x_1+x_2}{2}\right)+f\left(\frac{x_2+x_3}{2}\right)+\cdots+f\left(\frac{x_{n-1}+x_n}{2}\right)\right)$ *or* $w_1f\left(\frac{x_1+x_2}{2}\right)+w_2f\left(\frac{x_2+x_3}{2}\right)+\cdots+w_{n-1}f\left(\frac{x_{n-1}+x_n}{2}\right)$

Note: $w=\frac{b-a}{n}$ and applies only for equal sub intervals

14. $T_n=\frac{w}{2}(y_1+2y_2+\cdots+2y_{n-1}+y_n)$ **or** $\frac{1}{2}(w_1(y_1+y_2)+w_2(y_2+y_3)+...)$

20. Area $=\int_{x_{left}}^{x_{right}}\left[f(x)_{top}-f(x)_{down}\right]dx$ or Area $=\int_{y_{down}}^{y_{top}}\left[f(y)_{left}-f(y)_{right}\right]dy$

21. Vol of rev $=\pi\int_{x_{left}}^{x_{right}}\left\{\left[f(x)_{top}-a\right]^2-\left[f(x)_{down}-a\right]^2\right\}dx$ Vol Cross sect.$=\int_a^b A(x)dx$

ANTIDIFFERENTION (INTEGRATION) RULES

1. $\int x^n dx=\frac{x^{n+1}}{n+1}+C$ 2. $\int\frac{1}{x}dx=\ln|x|+C$ 3. $\int\frac{1}{ax+b}dx=\frac{\ln|ax+b|}{a}+C$ 4. $\int e^{kx}dx=\frac{e^{kx}}{k}+C$

5. $\int a^x dx=\frac{a^u}{\ln a}+C$ 6. $\int\sin kx\,dx=-\frac{\cos kx}{k}+C$ 7. $\int\cos kx\,dx=\frac{\sin kx}{k}+C$

8. $\int\sec x\tan x\,dx=\sec x+C$ 9. $\int\sec^2 x\,dx=\tan x+C$ 10. $\int\csc x\cot x\,dx=-\csc x+C$

15. **Average Value of f** $\text{av}(f)=f_{ave}=\frac{1}{b-a}\int_a^b f(x)dx$

16. FTC I: $\int_a^b f'(x)dx=f(b)-f(a)$

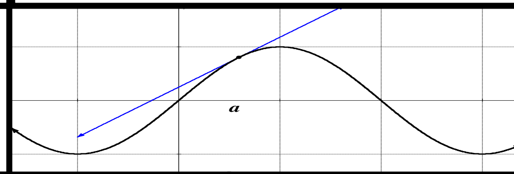
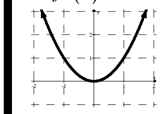
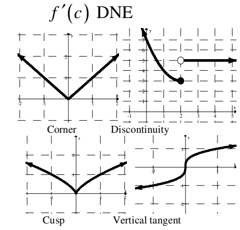
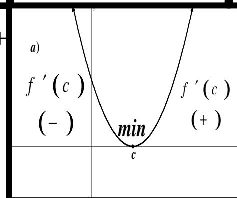
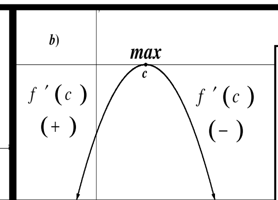
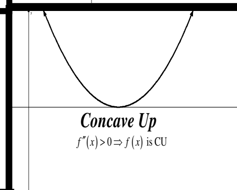
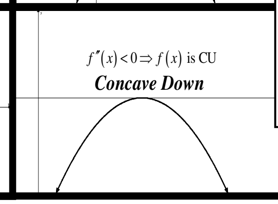
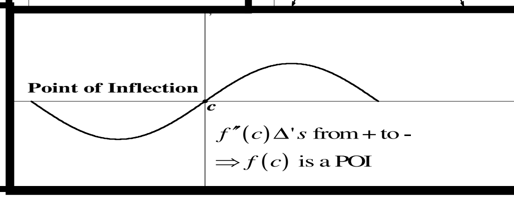
17 FTC II i) $\int_a^x f(t)dt=F(x)$

ii) $\frac{d}{dx}\int_a^x f(t)dt=f(x)$ iii) $\frac{d}{dx}\int_{h(x)}^{g(x)} f(t)dt=f(g(x))\cdot g'(x)-h(g(x))\cdot h'(x)$

18. Integration by parts: $\int v\,du=uv-\int v\,du$ use LIPET to select u

19. Integration by substitution: $\int f(g(x))g'(x)=\int f(u)du$

VOCABULARY (4)

Term	Verbal Description	Symbolic	Graphical
1. Derivative of f at a:	The instantaneous rate of change of the function at a or the slope of the tangent line at a	$f'(a)=\left.\frac{df}{dx}\right\|_{x=a}=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$	
2. Critical Number c	A number c in an open (a, b) interval where the derivative is zero or does not exist	$c\in(a,b)$ where $f'(c)=0$ or $f'(c)$ DNE	$f'(c)=0$  $f'(c)$ DNE 
3. First Derivative Test	a) If the first derivative changes from negative to positive at c then the function has a relative minimum at c b) If the first derivative changes from positive to negative at c then the function has a relative maximum at c	a) If $f'(c)$ Δ's from $-$ to $+$ $\Rightarrow f'(c)$ is a min b) If $f'(c)$ Δ's from $+$ to $-$ $\Rightarrow f'(c)$ is a max	 
4. Concavity Test	a) If the second derivative is positive on an interval I then the function is Concave Up on I b) If the second derivative is negative on an interval I the function is Councave down on I	a) If $f''(c)>0$ on I $\Rightarrow f(x)$ is CU on I b) If $f''(c)<0$ on I $\Rightarrow f(x)$ is CD on I	 
5. Point of Inflection at c	**f: Is a point where the concavity of f changes** **f': Is a point where f' changes from increasing to decreasing or decreasing to increasing** **f'': Is a point where f'' changes from positive to negative or negative to positive**	f Δ's from CU to CD or CD to CU f' Δ's from ↗ to ↘ or ↘ to ↗ $f''(x)$ Δ's from $+$ to $-$ or $-$ to	

<u>Motion definitions and Equations</u>

6. Displacement: A Vector quantity that represents the net change in position $s(t)=x(b)-x(a)=\int_a^b v(t)$

7. Distance: A scalar quantity that represents total movement regardless of sign $d(t)=\|x(b)-x(a)\|=\int_a^b \|v(t)\|dt$

8. Velocity: A Vector quantity that represents the rate of change of position $v(t)=s'(t)$

9. Speed: A scalar quantity that represents the rate of covering distance Speed $=\|v(t)\|$

10. Acceleration: A vector quantity that represents the rate of change of velocity $a(t)=v'(t)=s''(t)$

11. Given initial position $s(a)=C$ the final position is given by $s(b)=s(a)+\int_a^b s'(t)\,dt$

<u>Reciprocal</u>

$\sin x=\frac{1}{\csc x}$ $\csc x=\frac{1}{\sin x}$

$\cos x=\frac{1}{\sec x}$ $\sec x=\frac{1}{\cos x}$

$\tan x=\frac{1}{\cot x}$ $\cot x=\frac{1}{\tan x}$

<u>Quotient</u>

$\tan x=\frac{\sin x}{\cos x}$

$\cot x=\frac{\cos x}{\sin x}$

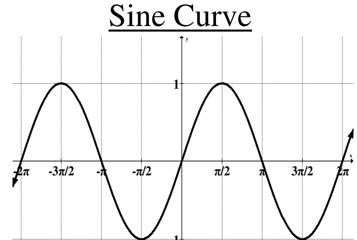
<u>Pythagorean</u>

$\sin^2 x+\cos^2 x=1$

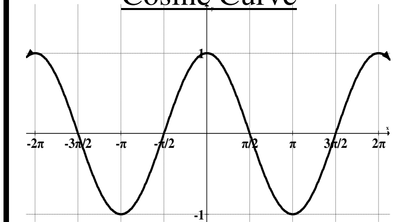
$\tan^2 x+1=\sec^2 x$

$\cot^2 x+1=\csc^2 x$

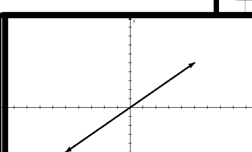
<u>Sine Curve</u>



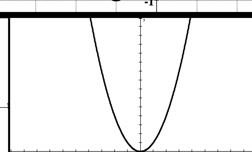
<u>Cosine Curve</u>



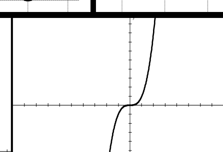
	0	$\pi/6$ (30°)	$\pi/4$ (45°)	$\pi/3$ (60°)	$\pi/2$ (90°)
$\sin x$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos x$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan x$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Und.
$\csc x$	Und.	2	$2/\sqrt{2}$	$2/\sqrt{3}$	1
$\sec x$	1	$2/\sqrt{3}$	$2/\sqrt{2}$	2	Und.
$\cot x$	Und.	$2/\sqrt{3}$	1	$1/\sqrt{3}$	0



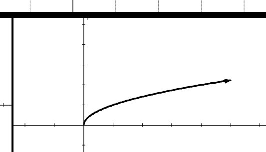
Linear $y=x$



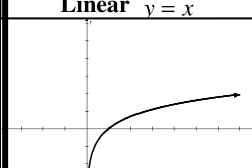
quadratic $y=x^2$



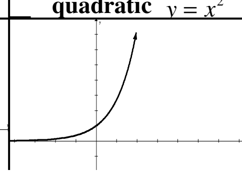
Cubic $y=x^3$



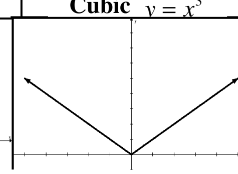
Radical $y=\sqrt{x}$



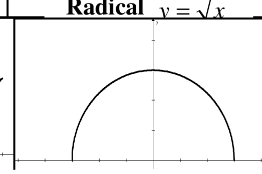
Logarithmic $y=\ln x$



Exponential $y=e^x$



Absolute value $y=\|x\|$



circular $y=\sqrt{9-x^2}$