Calculus AB Checklist of Skills

I. Functions, Graphs, and Limits

Limits of functions (including one-sided limits) … Chapter 2

- ☐ An intuitive understanding of the limiting process
- ☐ Calculate limits using algebra
- ☐ Estimate limits from graphs or tables of data

Asymptotic and unbounded behavior … Chapter 2

- ☐ Understand asymptotes in terms of graphical behavior
- ☐ Describing asymptotic behavior in terms of limits involving infinity
- ☐ Compare relative magnitudes of functions and their rates of change

Continuity as a property of functions … Chapter 2

- ☐ An intuitive understanding of continuity
- ☐ Understand continuity in terms of limits
- ☐ Geometric understanding of graphs of continuous functions (**I**ntermediate **V**alue **T**heorem)

II. Derivatives

Concept of the derivative … Chapter 3 part 1

- ☐ Derivative presented graphically, numerically, and analytically
- ☐ Derivative interpreted as an instantaneous rate of change
- ☐ Derivative defined as the limit of the difference quotient
- ☐ Relationship between differentiability and continuity

Derivative at a point … Chapter 3 part 1

- ☐ Slope of a curve at a point. Know when a derivative does not exist.
- ☐ Tangent line to a curve at a point and local linear approximation
- ☐ Instantaneous rate of change as the limit of average rate of change
- ☐ Approximate rate of change from graphs and tables of values

Computation of derivatives … Chapter 3 parts 1 & 2

- ☐ Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- ☐ Derivative rules for sums, products, and quotients of functions
- ☐ Chain rule and implicit differentiation

Derivative as a function … Chapter 4 part 1

- ☐ Know corresponding characteristics of graphs of f and f'
- ☐ Know relationship between the increasing and decreasing behavior of f and the sign of f'
- ☐ The **M**ean **V**alue **T**heorem and its geometric interpretation

Second derivatives … Chapter 4 part 1

- ☐ Know corresponding characteristics of the graphs of f, f', and f''
- ☐ Know relationship between the concavity of f and the sign of f''
- ☐ Points of inflection as places where concavity changes

Applications of derivatives …

- ☐ Analysis of curves, including the notions of monotonicity and concavity … **Ch 4 pt 1**
- ☐ Optimization, both absolute (global) and relative (local) extremes … **Ch 4 pt 1**
- ☐ Modeling rates of change, including related rates problems … **Ch 4 pt 2**
- ☐ Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration … **Ch 3 pt 1, Ch 4 pt 1**
- ☐ Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations … **Ch 6**

III. Integrals

Numerical approximations to definite integrals … Chapter 5

- Use of Riemann sums (Left, Right, and Midpoint evaluation points) and Trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

Interpretations and properties of definite integrals … Chapter 5

- Definite integral as a limit of Riemann sums
- Basic properties of definite integrals (examples include additivity and linearity)

Fundamental Theorem of Calculus … Chapter 5

- Use of the Fundamental Theorem to evaluate definite integrals
- Definite integral of the rate of change of a quantity interpreted as the net change in that quantity over the interval: $\int_a^b f(x)dx = f(b) - f(a)$ or $f(a) + \int_a^b f(x)dx = f(b)$

Techniques of Integration (anti-differentiation) … Chapter 5 & 6

- Anti-derivatives following directly from derivatives of basic functions
- Anti-derivatives by u-substitution of variables (including change of limits for definite integrals)

Applications of antidifferentiation … Chapter 6

- Find specific anti-derivative general solutions and particular solutions
- Solve separable differential equations and write the particular solution as a function.

Applications of integrals … Chapter 7

- Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. The emphasis is on setting up a definite integral. Specific applications:
  - to find the area of a region
  - the volume of a solid with known cross sections
  - the average value of a function … **Ch 5**
  - the distance traveled by a particle along a line
  - accumulated change from a rate of change

CALCULATOR functions:

- graph a function and adjust the window appropriately for analysis
- find specific function values, zeros, & intersection points
- Find the numerical derivative (at a point on a curve & as a graph)
  - **nderiv (*f (x)* , *x, a*)** to find *f '(a) at x = a*
  - **Y1 = nderiv(*f (x), x, x*)** to graph the numerical derivative function *f '(x)*
- Find the numerical definite integral **fnInt (*f (x), x, a, b*)**

PRIOR KNOWLEDGE (Just about everything from your high school career):

- Trigonometry function definitions and values from the Unit Circle
- Parent Functions (properties, domains, ranges) : $y = x^n$, $y = \sqrt[n]{x}$, $y = |x|$,

  $y = e^x$, $y = \ln x$, $y = \sin x$, $y = \cos x$, $y = \tan x$
- Geometry Areas: Rectangles, Triangles, Trapezoids, Circles, plus $A_{EQ\Delta} = \dfrac{x^2\sqrt{3}}{4}$
- Algebra: factoring, domain restrictions, exponent rules, inverse operations

1972 AB 1

Let $f(x) = 4x^3 - 3x - 1$.

a. Find the x-intercepts of the graph of f.
b. Write an equation for the tangent line to the graph of f at x = 2.
c. Write an equation of the graph that is the reflection across the y-axis of the graph of f.

1972 AB 2 BC 1

A particle starts at time t = 0 and moves on a number line so that its position at time t is given by $x(t) = (t - 2)^3 (t - 6)$.

a. When is the particle moving to the right?
b. When is the particle at rest?
c. When does the particle change direction?
d. What is the farthest to the left of the origin that the particle moves?

1972 AB 4 BC 3

A man has 340 yards of fencing for enclosing two separate fields, one of which is to be a rectangle twice as long as it is wide and the other a square. The square field must contain at least 100 square yards and the rectangular one must contain at least 800 square yards.

a. If x is the width of the rectangular field, what are the maximum and minimum possible values of x?
b. What is the greatest number of square yards that can be enclosed in the two fields? Justify your answer.

1972 AB 5

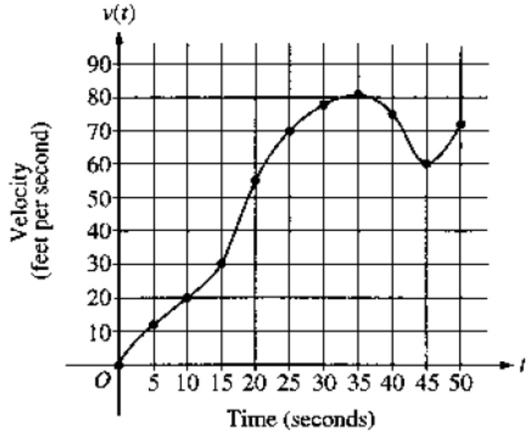
Let $y = 2e^{\cos(x)}$.

a. Calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.
b. If x and y both vary with time in such a way that y increases at a steady rate of 5 units per second, at what rate is x changing when $x = \frac{\pi}{2}$.

1973 AB 3 BC 1

Given the curve $x + xy + 2y^2 = 6$.

a. Find an expression for the slope of the curve at any point (x,y) on the curve.
b. Write an equation for the line tangent to the curve at the point (2,1).
c. Find the coordinate of all other points on this curve with slope equal to the slope at (2,1).



t (seconds)	v(t) (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

1998 AB 3

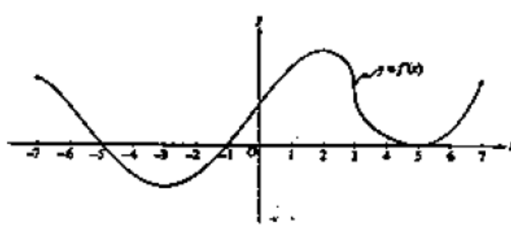
The graph of the velocity v(t), in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for v(t), at 5-second intervals of time t, is shown to the right of the graph.

a. During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

b. Find the average acceleration of the car, in ft/sec^2, over the interval $0 \leq t \leq 50$.

c. Find one approximation for the acceleration of the car in ft/sec^2, at t = 40. Show the computations you used to arrive at your answer.

d. Approximate the integral from 0 to 50 of v(t) dt with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.



2000 AB 3

The figure above shows the graph of f f', the derivative of the function f, for $-7 \leq x \leq 7$.

The graph of f' has horizontal tangent lines at x = -3, x = 2, and x = 5, and a vertical tangent line at x = 3.

a. Find all values of x, for $-7 \leq x \leq 7$, at which f attains a relative minimum. Justify your answer.

b. Find all values of x, for $-7 \leq x \leq 7$, at which f attains a relative maximum. Justify your answer.

c. Find all values of x, for $-7 \leq x \leq 7$, at which $f''(x) < 0$.

d. At what value of x, for $-7 \leq x \leq 7$, does f attain its absolute maximum? Justify your answer.