

## I. Functions, Graphs, and Limits

### Limits of functions (including one-sided limits) ...

- ┆ An intuitive understanding of the limiting process
- ┆ Calculate limits using algebra
- ┆ Estimate limits from graphs or tables of data

### Asymptotic and unbounded behavior ...

- ┆ Understand asymptotes in terms of graphical behavior
- ┆ Describing asymptotic behavior in terms of limits involving infinity
- ┆ Compare relative magnitudes of functions and their rates of change

### Continuity as a property of functions ...

- ┆ An intuitive understanding of continuity
- ┆ Understand continuity in terms of limits
- ┆ Geometric understanding of graphs of continuous functions (Intermediate Value Theorem)

## II. Derivatives

### Concept of the derivative ...

- ┆ Derivative presented graphically, numerically, and analytically
- ┆ Derivative interpreted as an instantaneous rate of change
- ┆ Derivative defined as the limit of the difference quotient
- ┆ Relationship between differentiability and continuity

### Derivative at a point ...

- ┆ Slope of a curve at a point. Know when a derivative does not exist.
- ┆ Tangent line to a curve at a point and local linear approximation
- ┆ Instantaneous rate of change as the limit of average rate of change
- ┆ Approximate rate of change from graphs and tables of values

### Computation of derivatives ...

- ┆ Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions
- ┆ Derivative rules for sums, products, and quotients of functions
- ┆ Chain rule and implicit differentiation

### Derivative as a function ...

- ┆ Know corresponding characteristics of graphs of  $f$  and  $f'$
- ┆ Know relationship between the increasing and decreasing behavior of  $f$  and the sign of  $f'$
- ┆ The Mean Value Theorem and its geometric interpretation

### Second derivatives ...

- ┆ Know corresponding characteristics of the graphs of  $f$ ,  $f'$ , and  $f''$
- ┆ Know relationship between the concavity of  $f$  and the sign of  $f''$
- ┆ Points of inflection as places where concavity changes

### Applications of derivatives ...

- ┆ Analysis of curves, including the notions of monotonicity and concavity ...
- ┆ Optimization, both absolute (global) and relative (local) extremes ...
- ┆ Modeling rates of change, including related rates problems ...
- ┆ Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration ...
- ┆ Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations ...
- ┆ Use L'Hopital's rule to find limits with indeterminate forms

### III. Integrals

#### Numerical approximations to definite integrals ...

- ┆ Use of Riemann sums (Left, Right, and Midpoint evaluation points) and Trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

#### Interpretations and properties of definite integrals ...

- ┆ Definite integral as a limit of Riemann sums
- ┆ Basic properties of definite integrals (examples include additivity and linearity)

#### Fundamental Theorem of Calculus ...

- ┆ Use of the Fundamental Theorem to evaluate definite integrals
- ┆ Definite integral of the rate of change of a quantity interpreted as the net change in that quantity over the

interval: 
$$\int_a^b f(x)dx = f(b) - f(a) \quad \text{or} \quad f(a) + \int_a^b f(x)dx = f(b)$$

#### Techniques of Integration (anti-differentiation) ...

- ┆ Anti-derivatives following directly from derivatives of basic functions
- ┆ Anti-derivatives by u-substitution of variables (including change of limits for definite integrals)

#### Applications of antidifferentiation ...

- ┆ Find specific anti-derivative general solutions and particular solutions
- ┆ Solve separable differential equations and write the particular solution as a function.

#### Applications of integrals ...

- ┆ Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. The emphasis is on setting up a definite integral. Specific applications:
  - ┆ to find the area of a region
  - ┆ the volume of a solid with known cross sections
  - ┆ the average value of a function ...
  - ┆ the distance traveled by a particle along a line
  - ┆ accumulated change from a rate of change

### CALCULATOR functions:

- ┆ graph a function and adjust the window appropriately for analysis
- ┆ find specific function values, zeros, & intersection points
- ┆ Find the numerical derivative (at a point on a curve & as a graph)
  - **nderiv** (  $f(x)$ ,  $x$ ,  $a$  ) to find  $f'(a)$  at  $x = a$
  - **Y1 = nderiv**(  $f(x)$ ,  $x$ ,  $x$  ) to graph the numerical derivative function  $f'(x)$
- ┆ Find the numerical definite integral **fnInt** (  $f(x)$ ,  $x$ ,  $a$ ,  $b$  )

### PRIOR KNOWLEDGE (Just about everything from your high school career):

- ┆ Trigonometry function definitions and values from the Unit Circle
- ┆ Parent Functions (properties, domains, ranges):  $y = x^n$ ,  $y = \sqrt[n]{x}$ ,  $y = |x|$ ,  
 $y = e^x$ ,  $y = \ln x$ ,  $y = \sin x$ ,  $y = \cos x$ ,  $y = \tan x$
- ┆ Geometry Areas: Rectangles, Triangles, Trapezoids, Circles, plus  $A_{EQ\Delta} = \frac{r^2\sqrt{3}}{4}$
- ┆ Algebra: factoring, domain restrictions, exponent rules, inverse operations

**1972 AB 1**

Let  $f(x) = 4x^3 - 3x - 1$ .

- Find the x-intercepts of the graph of  $f$ .
- Write an equation for the tangent line to the graph of  $f$  at  $x = 2$ .
- Write an equation of the graph that is the reflection across the y-axis of the graph of  $f$ .

**1972 AB 2 BC 1**

A particle starts at time  $t = 0$  and moves on a number line so that its position at time  $t$  is given by  $x(t) = (t - 2)^3 (t - 6)$ .

- When is the particle moving to the right?
- When is the particle at rest?
- When does the particle change direction?
- What is the farthest to the left of the origin that the particle moves?

**1972 AB 4 BC 3**

A man has 340 yards of fencing for enclosing two separate fields, one of which is to be a rectangle twice as long as it is wide and the other a square. The square field must contain at least 100 square yards and the rectangular one must contain at least 800 square yards.

- If  $x$  is the width of the rectangular field, what are the maximum and minimum possible values of  $x$ ?
- What is the greatest number of square yards that can be enclosed in the two fields? Justify your answer.

**1972 AB 5**

Let  $y = 2e^{\cos(x)}$ .

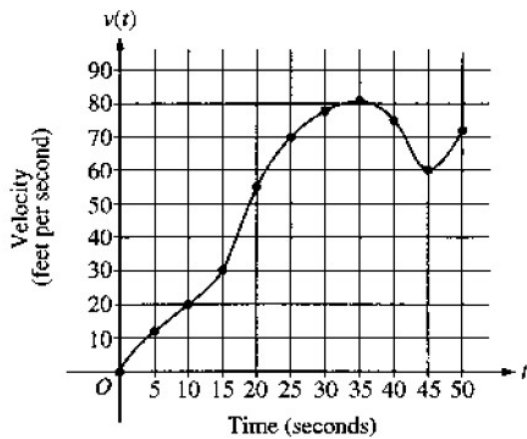
- Calculate  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ .
- If  $x$  and  $y$  both vary with time in such a way that  $y$  increases at a steady rate of 5 units per second, at what rate is  $x$  changing when  $x = \frac{\pi}{2}$ .

**1973 AB 3 BC 1**

Given the curve  $x + xy + 2y^2 = 6$ .

- Find an expression for the slope of the curve at any point  $(x, y)$  on the curve.
- Write an equation for the line tangent to the curve at the point  $(2, 1)$ .
- Find the coordinate of all other points on this curve with slope equal to the slope at  $(2, 1)$ .

UCLA AP READINESS FRQ PRACTICE- Comprehensive Review

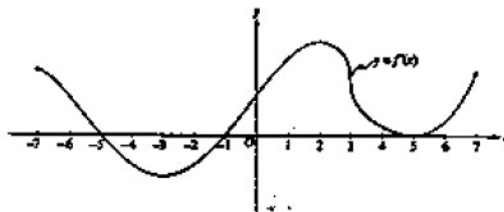


$t$ (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

**1998 AB 3**

The graph of the velocity  $v(t)$ , in ft/sec, of a car traveling on a straight road, for  $0 \leq t \leq 50$ , is shown above. A table of values for  $v(t)$ , at 5-second intervals of time  $t$ , is shown to the right of the graph.

- During what intervals of time is the acceleration of the car positive? Give a reason for your answer.
- Find the average acceleration of the car, in  $\text{ft}/\text{sec}^2$ , over the interval  $0 \leq t \leq 50$ .
- Find one approximation for the acceleration of the car in  $\text{ft}/\text{sec}^2$ , at  $t = 40$ . Show the computations you used to arrive at your answer.
- Approximate the integral from 0 to 50 of  $v(t) dt$  with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.



**2000 AB 3**

The figure above shows the graph of  $f'$ , the derivative of the function  $f$ , for  $-7 \leq x \leq 7$ .

The graph of  $f'$  has horizontal tangent lines at  $x = -3$ ,  $x = 2$ , and  $x = 5$ , and a vertical tangent line at  $x = 3$ .

- Find all values of  $x$ , for  $-7 \leq x \leq 7$ , at which  $f$  attains a relative minimum. Justify your answer.
- Find all values of  $x$ , for  $-7 \leq x \leq 7$ , at which  $f$  attains a relative maximum. Justify your answer.
- Find all values of  $x$ , for  $-7 \leq x \leq 7$ , at which  $f''(x) < 0$ .
- At what value of  $x$ , for  $-7 \leq x \leq 7$ , does  $f$  attain its absolute maximum? Justify your answer.