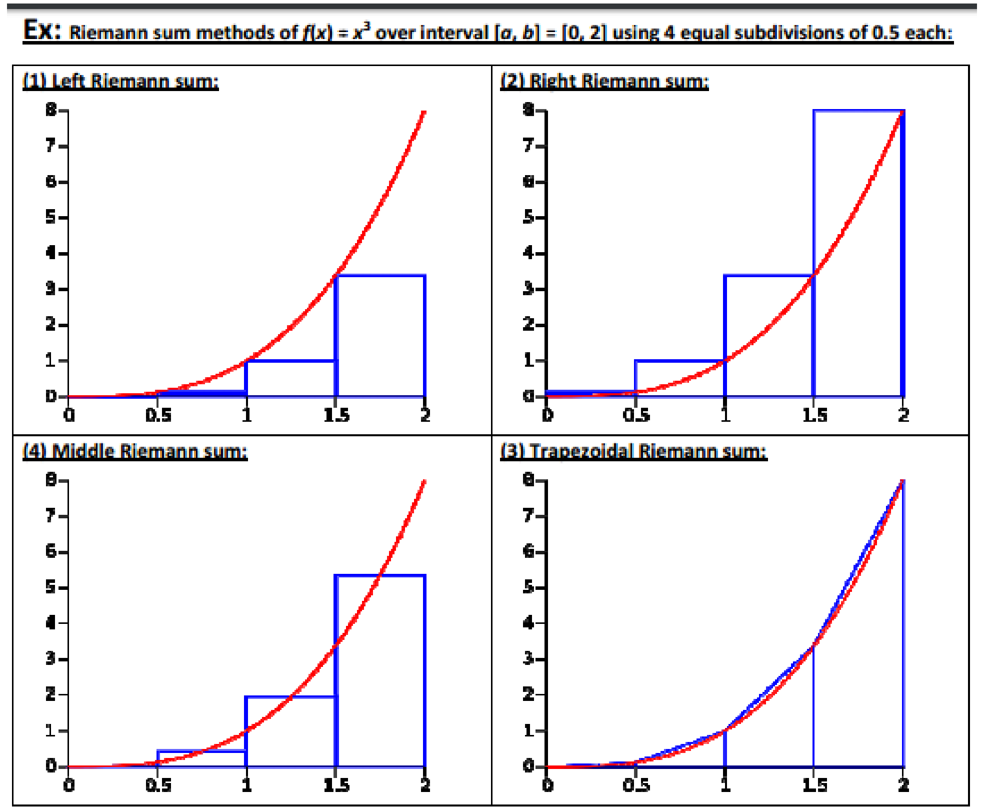
AP Readiness- Riemann Sums Revisited, AP “Table” Problems

The AP Calculus exams include multiple-choice and free-response questions in which the stem of the question includes a table of numerical information from which you are asked questions about the function, its graph, its derivative, or its definite integral. The answers were usually approximations.

Sometimes a function that “modeled” the function in the table was also given and you may be asked similar questions based on the model.

Explanations of what was found were also required in some questions. Thus, starting with a numerical prompt, numerical, graphical, analytic and verbal replies were required.



What you should know how to do:

Here are the most common things that are asked on AP Calculus exam table problems:

1. Approximate a derivative (slope, rate of change, average rate of change) using difference quotients.
2. Use a Riemann sum or a Trapezoidal approximation to approximate a definite integral
3. Explain the meaning of a definite integral in the context of the problem.
4. Calculate a tangent line approximation (local linear approximation).
5. Give the units of the answer (unit analysis).
6. Answer theory questions usually related to the Mean Value Theorem (MVT), Rolle’s Theorem, the Intermediate Value Theorem (IVT) or the Extreme Value Theorem (EVT).
7. Give information about the graph of the function.
8. One thing not to do: Do not use your graphing calculators to produce a regression equation and use that to answer the questions. Finding regression equations, while very good mathematics, is not one of the four things you are allowed to do on the exam with your calculator. Also, a regression model is not the given function, only an approximation of it. You are expected to work from the table using calculus techniques and approximations. Using a regression will not earn any points.

*from Lin McMullin

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MULTIPLE CHOICE

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

A table of values for a continuous function f is shown above. If four equal subintervals of $[0,2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

x	2	5	7	8
$f(x)$	10	30	40	20

The function f is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

a. 110

b. 130

c. 160

d. 190

e. 210

t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

a. 26 ft/sec

b. 30 ft/sec

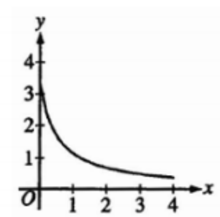
c. 37 ft/sec

d. 39 ft/sec

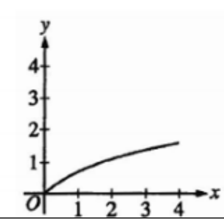
e. 41 ft/sec

2003 #85 (BC) - Calc OK: If a trapezoidal sum overapproximates $\int_0^4 f(x)dx$ and a right Riemann sum underapproximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?

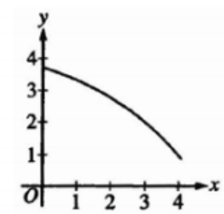
a.



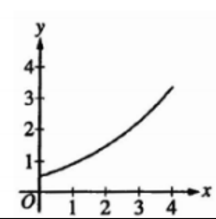
b.



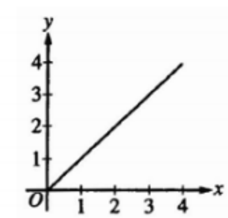
c.



d.



e.



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Let $y(t)$ represent the population of a town over a 20-year period, where y is a differentiable function of t. The table below shows the population recorded at selected times.

t (yrs)	0	4	10	13	20
$y(t)$ (people)	2500	2724	3108	3697	4283

1. Use data from the table to find an approximation for $y'(12)$, and explain the meaning of $y'(12)$ in terms of the population of the town. Show the computations that lead to your answer.

2. Use data from the table and a trapezoidal approximation with four subintervals to approximate the average population of the town over the 20-year period. Show the computations that lead to your answer.

The rate at which water flows into a tank, in gallons per hour, is given by a positive continuous function R of time t. The table below shows the rate at selected values of t for a 12-hour period.

t (hrs)	0	2	4	6	8	10	12
$R(t)$(gal/hr)	12.5	13.4	13.9	14.3	14.6	14.8	14.7

1. Use a midpoint Riemann sum with three subintervals to approximate:

$$\int_0^{12} R(t)\,dt,$$

and explain the meaning of this definite integral in terms of the water flow, using correct units. Show the computations that lead to your answer.

2. A model for the rate of water flow is given by the function:

$$P(t) = \frac{1}{60}\left(750 + 24t - t^2\right)$$

where the positive rate P is measured in gallons per hour and the time t is measured in hours. Use $P(t)$ to find the average rate of water flow during the 12-hour time period. Indicate units of measure.

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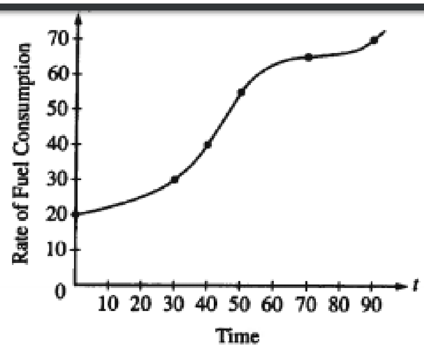

t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^b R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?