**LIMIT LAWS**

1. $\lim_{x\to 0}\frac{\sin x}{x}=1$ 2. $\lim_{x\to\infty}\frac{\sin x}{x}=0$ 3. $\lim_{x\to 0}\frac{\sin^2 x}{x}=0$

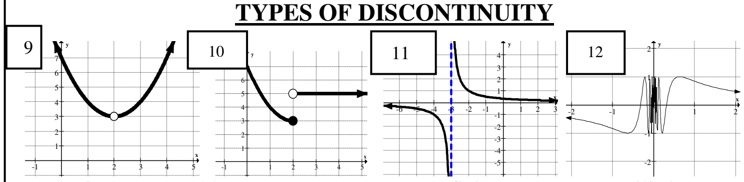
4. $\lim_{x\to 0}\frac{\cos x-1}{x}=0$ 5. $\lim_{x\to 0}\frac{\sin(ax)}{(bx)}=\lim_{x\to 0}\frac{\sin(ax)}{\sin(bx)}=\lim_{x\to 0}\frac{(bx)}{\sin(ax)}=\frac{a}{b}$

6. $\lim_{x\to a} f(x)=L$ (exists) If and only if $\lim_{x\to a^+} f(x)=\lim_{x\to a^-} f(x)=L$

7. $f(x)$ is cont $a$ if $\lim_{x\to a} f(x)=f(a)$

8. **Continuity at** $a$ **if** $\lim_{x\to a^-} f(x)=f(a)=\lim_{x\to a^-} f(x)$

**TYPES OF DISCONTINUITY**




Point or removable | Jump | Infinite | Oscillating

**ARITHMETIC OF INFINITY**

| | |
|---|---|
| 1. $\infty+\infty=\infty$ 2. $n+\infty=\infty$ 3. $\infty+0=\infty$ 4. $0+\infty=\infty$ } (+) | 1. $\infty\bullet\infty=\infty$ 2. $n\bullet\infty=\infty$ 3. $0\bullet\infty=0$ } (×) |
| 1. $\infty-\infty=und$ 2. $n-\infty=-\infty$ 3. $\infty-n=\infty$ 4. $n-n^-=0^-=-\frac{1}{\infty}$ 5. $n^+-n=0^+=\frac{1}{\infty}$ } (−) | 1. $\infty/\infty=und$ 2. $n/\pm\infty=0$ 3. $\infty/n=\infty$ 4. $n/0=\pm\infty$ 5. $n/0^+=\infty$ 6. $n/0^-=-\infty$ } (÷) |
| 1. $\infty^\infty=\infty$ 2. $\infty^n=\infty$ 3. $\infty^0=1$ 5. $0^\infty=0$ 4. $n^\infty=\infty$ } power | 1. $-1\le\sin(\pm\infty)\le 1$ 2. $-1\le\cos(\pm\infty)\le 1$ } Trig |

LIMITS(1)

**Basic Rules**

1. $\frac{d}{dx}c=0$ (Constant) 2. $\frac{d}{dx}c[f(x)]=c\frac{d}{dx}f(x)$ (constant multiple)

3. $\frac{d}{dx}x^n=nx^{n-1}$ (power) 4. $\frac{d}{dx}(u\pm v)=\frac{d}{dx}u\pm\frac{d}{dx}v$ (Sum & Difference)

5. $\frac{d}{dx}(uv)=v\frac{d}{dx}u+u\frac{d}{dx}v$ (Product) 6. $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{d}{dx}u-u\frac{d}{dx}v}{v^2}$ (Quotient)

7. $(f^{-1})'(x)=\frac{1}{f'(f^{-1}(x))}$ (Inverse)

8. $\frac{d}{dx}f(g(x))=\frac{d}{dx}f(u)\bullet\frac{d}{dx}g(x)$ where $u=g(x)$ (Quotient)

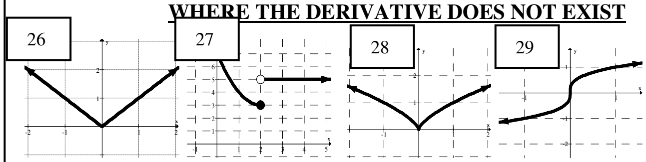
**Exponential and Logarithmic Functions**

20. $\frac{d}{dx}\ln u=\frac{1}{u}\frac{du}{dx}$ 21. $\frac{d}{dx}\log_a u=\frac{1}{u\ln a}\frac{du}{dx}$

22. $\frac{d}{dx}e^u=e^u\frac{du}{dx}$ 23. $\frac{d}{dx}a^u=a^u\ln a\frac{du}{dx}$

24. $\frac{d}{dx}\sqrt{u}=\frac{1}{2\sqrt{u}}\frac{du}{dx}$ 25. $\frac{d}{dx}|u|=\frac{u}{|u|}\frac{du}{dx}$

**WHERE THE DERIVATIVE DOES NOT EXIST**




Corner | Discontinuity | Cusp | Vertical tangent

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Linearization: $L(x)=f(a)+f'(a)(x-a)$

**Trigonometric Functions**

8. $\frac{d}{dx}\sin u=\cos u\frac{du}{dx}$ 9. $\frac{d}{dx}\cos u=-\sin u\frac{du}{dx}$

10. $\frac{d}{dx}\tan u=\sec^2 u\frac{du}{dx}$ 11. $\frac{d}{dx}\cot u=-\csc^2 u\frac{du}{dx}$

12. $\frac{d}{dx}\sec u=\sec u\tan u\frac{du}{dx}$ 13. $\frac{d}{dx}\csc u=-\csc u\cot u\frac{du}{dx}$

**Inverse Trigonometric Functions**

14. $\frac{d}{dx}\sin^{-1}u=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ 15. $\frac{d}{dx}\cos^{-1}u=\frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$

16. $\frac{d}{dx}\tan^{-1}u=\frac{1}{1+u^2}\frac{du}{dx}$ 17. $\frac{d}{dx}\cot^{-1}u=\frac{-1}{1+u^2}\frac{du}{dx}$

18. $\frac{d}{dx}\sec^{-1}u=\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$ 19. $\frac{d}{dx}\csc^{-1}u=\frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

DERITAVITIVES(2)

30. Mean Value Theorem: If $f$ is cont on $[a,b]$ on and diff on $(a,b)$ $\Rightarrow$ exist a $c\in(a,b)$ s.t. $f'(c)=\frac{f(b)-f(a)}{b-a}$

31. Rolle's Theorem: MVT where $f'(c)=\frac{f(b)-f(a)}{b-a}=0$

32. IntermediateValueTheorem.: If $f$ is cont on $[a,b]$ and $d\in[f(a),f(b)]$ then there is a $c\in[a,b]$ st $f(c)=d$

**Definition of Derivative**

33. $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

34. $f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

35. $f'(x)\approx$ **Average rate of change = Slope of Secant line** $=\frac{f(b)-f(a)}{b-a}$

**APPROXIMATING AREA**

11. LRAM$_n=w\left(f(x_1)+f(x_2)+..+f(x_{n-1})\right)$ **or** $w_1f(x_1)+w_2f(x_2)+..+w_{n-1}f(x_{n-1})$

12. RRAM$_n=w\left(f(x_2)+f(x_3)+\cdots+f(x_n)\right)$ **or** $w_1f(x_2)+w_2f(x_3)+..+w_{n-1}f(x_n)$

13. MRAM$_n=w\left(f\left(\frac{x_1+x_2}{2}\right)+f\left(\frac{x_2+x_3}{2}\right)+\cdots+f\left(\frac{x_{n-1}+x_n}{2}\right)\right)$ *or* $w_1f\left(\frac{x_1+x_2}{2}\right)+w_2f\left(\frac{x_2+x_3}{2}\right)+\cdots+w_{n-1}f\left(\frac{x_{n-1}+x_n}{2}\right)$

Note: $w=\frac{b-a}{n}$ and applies only for equal sub intervals

14. $T_n=\frac{w}{2}(y_1+2y_2+\cdots+2y_{n-1}+y_n)$ **or** $\frac{1}{2}\left(w_1(y_1+y_2)+w_2(y_2+y_3)+...\right)$

20. Area $=\int_{x_{left}}^{x_{right}}\left[f(x)_{top}-f(x)_{down}\right]dx$ or Area $=\int_{y_{down}}^{y_{top}}\left[f(y)_{left}-f(y)_{right}\right]dy$

21. Vol of rev $=\pi\int_{x_{left}}^{x_{right}}\left\{\left[f(x)_{top}-a\right]^2-\left[f(x)_{down}-a\right]^2\right\}dx$ Vol Cross sect.$=\int_a^b A(x)\,dx$

**ANTIDIFFERENTION (INTEGRATION) RULES**

1. $\int x^n dx=\frac{x^{n+1}}{n+1}+C$ 2. $\int\frac{1}{x}dx=\ln|x|+C$ 3. $\int\frac{1}{ax+b}dx=\frac{\ln|ax+b|}{a}+C$ 4. $\int e^{kx}dx=\frac{e^{kx}}{k}+C$

5. $\int a^x dx=\frac{a^u}{\ln a}+C$ 6. $\int\sin kx\,dx=-\frac{\cos kx}{k}+C$ 7. $\int\cos kx\,dx=\frac{\sin kx}{k}+C$

8. $\int\sec x\tan x\,dx=\sec x+C$ 9. $\int\sec^2 x\,dx=\tan x+C$ 10. $\int\csc x\cot x\,dx=-\csc x+C$

15. **Average Value of f** av$(f)=f_{ave}=\frac{1}{b-a}\int_a^b f(x)\,dx$

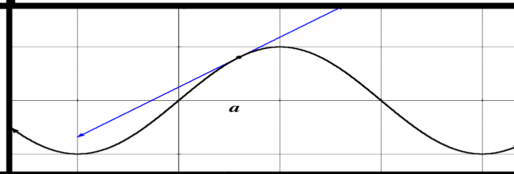
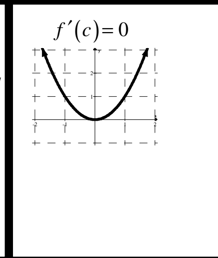
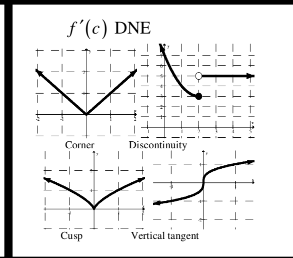
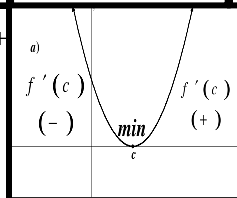
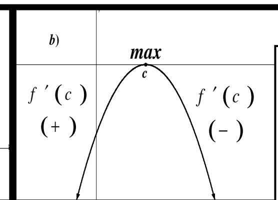
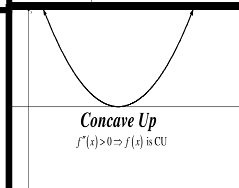
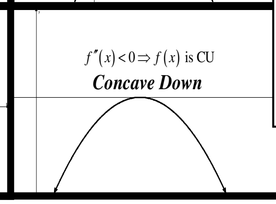
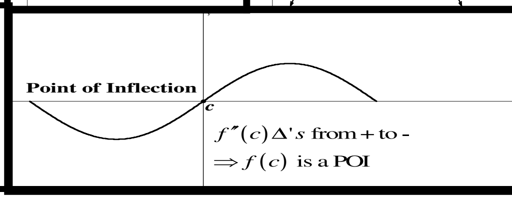
16. FTC I: $\int_a^b f'(x)\,dx=f(b)-f(a)$ 17 FTC II i) $\int_a^x f(t)\,dt=F(x)$

ii) $\frac{d}{dx}\int_a^x f(t)\,dt=f(x)$ iii) $\frac{d}{dx}\int_{h(x)}^{g(x)}f(t)\,dt=f(g(x))\cdot g'(x)-h(g(x))\cdot h'(x)$

18. Integration by parts: $\int v\,du=uv-\int v\,du$ use LIPET to select $u$

19. Integration by substitution: $\int f(g(x))g'(x)=\int f(u)\,du$

INTEGRALS (3)

| Term | Verbal Description | Symbolic | Graphical |
|---|---|---|---|
| 1. Derivative of $f$ at $a$: | The instantaneous rate of change of the function at $a$ or the slope of the tangent line at $a$ | $f'(a)=\left.\frac{df}{dx}\right\|_{x=a} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ |   |
| 2. Critical Number $c$ | A number $c$ in an open $(a, b)$ interval where the derivative is zero or does not exist | $c\in(a,b)$ where $f'(c)=0$ or $f'(c)$ DNE |     |
| 3. First Derivative Test | a) If the first derivative changes from negative to positive at c then the function has a relative minimum at c<br>b) If the first derivative changes from positive to negative at c then the function has a relative maximum at c | a) If $f'(c)$ $\Delta$'s from $-$ to $+$ $\Rightarrow f(c)$ is a min<br>b) If $f'(c)$ $\Delta$'s from $+$ to $-$ $\Rightarrow f(c)$ is a max |     |
| 4. Concavity Test | a) If the second derivative is positive on an interval I then the function is Concave Up on I<br>b) If the second derivative is negative on an interval I the function is Councave down on $I$ | a) If $f''(c)>0$ on $I$ $\Rightarrow f(x)$ is CU on $I$<br>b) If $f''(c)<0$ on $I$ $\Rightarrow f(x)$ is CD on $I$ |     |
| 5. Point of Inflection at $c$ | ***f*: Is a point where the concavity of f changes**<br>***f'*: Is a point where *f'* changes from increasing to decreasing or decreasing to increasing**<br>***f''*: Is a point where *f''* changes from positive to negative or negative to positive** | $f$ $\Delta$'s from CU to CD or CD to CU<br>$f'$ $\Delta$'s from ↗ to ↘ or ↘ to ↗<br>$f''(x)$ $\Delta$'s from + to − or − to |   |

VOCABULARY (4)

<u>Motion definitions and Equations</u>

6. Displacement: A Vector quantity that represents the net change in position $s(t)=x(b)-x(a)=\int_a^b v(t)$

7. Distance: A scalar quantity that represents total movement regardless of sign $d(t)=\|x(b)-x(a)\|=\int_a^b \|v(t)\|dt$

8. Velocity: A Vector quantity that represents the rate of change of position $v(t)=s'(t)$

9. Speed: A scalar quantity that represents the rate of covering distance Speed $=\|v(t)\|$

10. Acceleration: A vector quantity that represents the rate of change of velocity $a(t)=v'(t)=s''(t)$

11. Given initial position $s(a)=C$ the final position is given by $s(b)=s(a)+\int_a^b s'(t)\,dt$

**<u>Reciprocal</u>**

$\sin x=\frac{1}{\csc x}$ $\csc x=\frac{1}{\sin x}$

$\cos x=\frac{1}{\sec x}$ $\sec x=\frac{1}{\cos x}$

$\tan x=\frac{1}{\cot x}$ $\cot x=\frac{1}{\tan x}$

**<u>Quotient</u>**

$\tan x=\frac{\sin x}{\cos x}$

$\cot x=\frac{\cos x}{\sin x}$

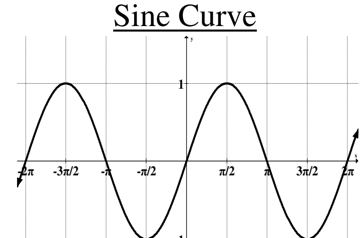
**<u>Pythagorean</u>**

$\sin^2 x+\cos^2 x=1$

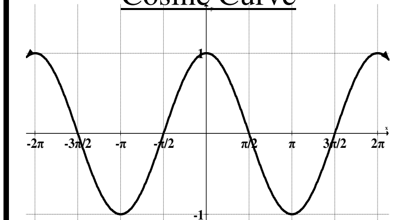
$\tan^2 x+1=\sec^2 x$

$\cot^2 x+1=\csc^2 x$

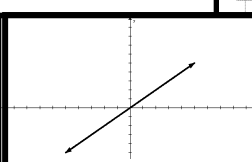
<u>Sine Curve</u>



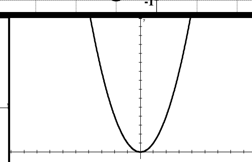

<u>Cosine Curve</u>



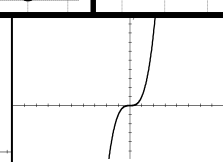

| | 0 | $\pi/6$ (30°) | $\pi/4$ (45°) | $\pi/3$ (60°) | $\pi/2$ (90°) |
|---|---|---|---|---|---|
| $\sin x$ | 0 | $1/2$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ | 1 |
| $\cos x$ | 1 | $\sqrt{3}/2$ | $\sqrt{2}/2$ | $1/2$ | 0 |
| $\tan x$ | 0 | $1/\sqrt{3}$ | 1 | $\sqrt{3}$ | Und. |
| $\csc x$ | Und. | 2 | $2/\sqrt{2}$ | $2/\sqrt{3}$ | 1 |
| $\sec x$ | 1 | $2/\sqrt{3}$ | $2/\sqrt{2}$ | 2 | Und. |
| $\cot x$ | Und. | $2/\sqrt{3}$ | 1 | $1/\sqrt{3}$ | 0 |



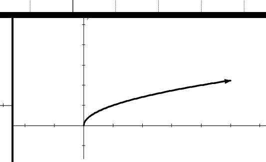
**Linear** $y=x$



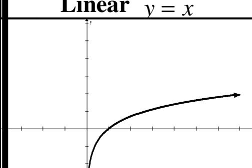
**quadratic** $y=x^2$



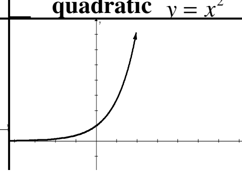
**Cubic** $y=x^3$



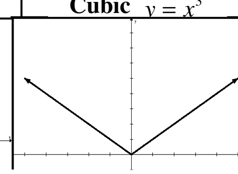
**Radical** $y=\sqrt{x}$



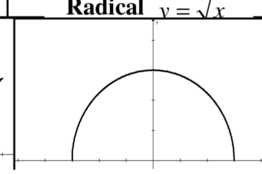
**Logarithmic** $y=\ln x$



**Exponential** $y=e^x$



**Absolute value** $y=|x|$



**circular** $y=\sqrt{9-x^2}$