calculus

TOPIC: *Evaluating Limits*

1. Remember to **ALWAYS** try direct substitution first.

2. Remember that in order for a limit to exist, $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)$

3. If direct substitution results in $\frac{0}{0}$ or $\frac{\infty}{\infty}$, do one of the following:

   - Use L'Hopital's Rule to evaluate the limit – that is, $\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$

   - Try an algebraic simplification such as factoring and canceling, multiplying by the conjugate, etc., to simplify the fraction

4. If direct substitution results in $\frac{0}{k}$, where k is any nonzero number, the limit = 0.

5. If direct substitution results in $\frac{k}{0}$, where k is any nonzero number, the limit DOES NOT EXIST.

TOPIC: *Continuity*

1. In order for a function to be continuous at some point c, the following three statements MUST BE TRUE (and you MUST show all three steps for credit on the AP Test):

   - □ $f(c)$ must exist
   - □ $\lim_{x \to c} f(x)$ must exist
   - □ **HOMERUN STATEMENT**: $\lim_{x \to c} f(x) = f(c)$

2. Remember that continuity does not imply differentiability (that is, if a function is continuous, that does not necessarily mean that it is differentiable…e.g., a continuous curve with a cusp would not be differentiable at the cusp).

3. Piecewise-defined functions tend to be the subject of many continuity questions; the key to showing continuity in these cases will be showing that the limits of each piece will match up and be equal (the limit of the function must exist at the 'pivot' point)

TOPIC: *The Difference Quotient*

1. Remember that $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ is used to find the derivative of the function $f(x)$; that is,

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=f'(x)$$

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2. You can evaluate these limits using L'Hopital's Rule…just remember to differentiate the top and bottom with respect to "*h*" and treat *x* as a constant.

For example, $\lim_{h\to 0}\frac{\ln(x+h)-\ln x}{h}=\frac{0}{0}\ \rightarrow\ \lim_{h\to 0}\frac{\frac{1}{x+h}-0}{1}=\frac{1}{x}$

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3. If the limit has numbers in it instead of *x*, you still evaluate the derivative with respect to *x* (as if there were an *x* there), and then plug the number in for *x*. For example:

$\lim_{h\to 0}\frac{\ln(3+h)-\ln 3}{h}$ really is going to compute the derivative of $\ln x$ at $x=3$

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TOPIC: *The Alternative Difference Quotient*

1. Remember that $\lim_{x \to c} \frac{f(x) - f(c)}{x - c} = f'(c)$. This form of the difference quotient is used to compute the derivative of a $f(x)$ at $x = c$.

   For example, $\lim_{x \to 3} \frac{(x-4)^3 - (-1)}{x - 3} = \frac{d}{dx}(x-4)^3 = 3(x-4)^2$, evaluated at $x = 3$ (which would lead to a final answer of $3(3-4)^2 = 3$.

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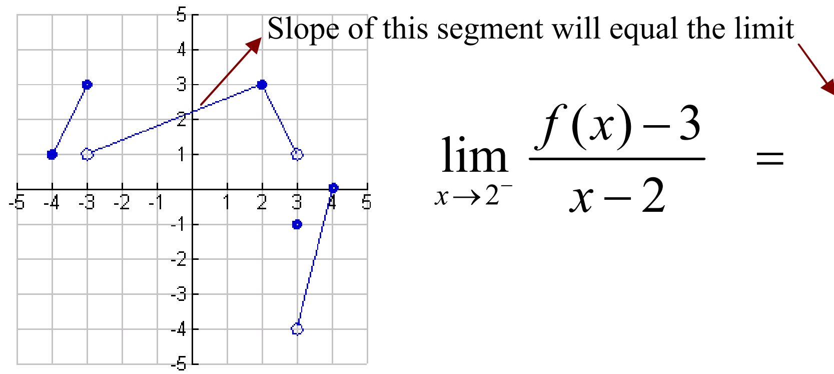
2. This limit is also used to find the **slope** of a curve as you approach specific *x* value on a function.



$$\lim_{x \to 2^-} \frac{f(x) - 3}{x - 2} = \frac{2}{5}$$

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3. You can also evaluate this limit using L'Hoptial's Rule.

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TOPIC: *Derivatives You Must Know*

1. $\frac{d}{dx}(c)=0$
2. $\frac{d}{dx}\left(u^n\right)=n\cdot u^{n-1}\cdot u'$
3. $\frac{d}{dx}(u\cdot v)=uv'+vu'$
4. $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vu'-uv'}{v^2}$
5. $\frac{d}{dx}\left[f(u)\right]=f'(u)\cdot u'$
6. $\frac{d}{dx}(\sin u)=\cos u\cdot u'$
7. $\frac{d}{dx}(\cos u)=-\sin u\cdot u'$
8. $\frac{d}{dx}(\tan u)=\sec^2 u\cdot u'$
9. $\frac{d}{dx}(\cot u)=-\csc^2 u\cdot u'$
10. $\frac{d}{dx}(\sec u)=\sec u\tan u\cdot u'$
11. $\frac{d}{dx}(\csc u)=-\csc u\cot u\cdot u'$
12. $\frac{d}{dx}(\ln u)=\frac{u'}{u}$
13. $\frac{d}{dx}(\log_a u)=\frac{1}{\ln a}\cdot\frac{u'}{u}$
14. $\frac{d}{dx}\left(e^u\right)=e^u\cdot u'$
15. $\frac{d}{dx}\left(a^u\right)=\ln a\cdot a^u\cdot u'$
16. $\frac{d}{dx}\left(a^u\right)=\ln a\cdot a^u\cdot u'$
17. $\frac{d}{dx}(\arcsin u)=\frac{u'}{\sqrt{1-u^2}}$
18. $\frac{d}{dx}(\arctan u)=\frac{u'}{1+u^2}$
19. $\frac{d}{dx}(\operatorname{arc\,sec} u)=\frac{u'}{|u|\sqrt{u^2-1}}$

TOPIC: *Implicit Differentiation*

1. Implicit differentiation is essentially an application of the CHAIN RULE. Pay attention to the variable which the derivative is being taken with respect to:

Example 1: $\frac{d}{dx}\left(y^3\right) = 3y^2 \cdot y' \quad or \quad 3y^2 \cdot \frac{dy}{dx}$

Example 2: $\frac{d}{dx}\left(x^3 + 5y^4\right) = 3x^2 + 20y^3 \cdot y'$

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2. Make sure that if you use the TI-89 to do implicit differentiation, place a "*" between *x* and *y* for any *xy* terms, etc. Otherwise, the calculator will compute the wrong derivative (it will assume *xy* is a single variable)

TOPIC: *Related Rates*

1. Related rates word problems are going to employ the use of IMPLICIT DIFFERENTIATION with respect to time, *t*, in the solving of their problems.

2. Remember the acronyms we have used:
   - GFE: **G**iven, **F**ind, **E**quation
   - DREDS: **D**rawing, **R**ates, **E**quation, **D**erivatives, **S**ubstitute Variables

3. Remember to always ask yourself, **"What's changing in the problem?"** Anything that 'changes' gets a variable (and consequently will get a derivative/differential term)
   - For example, if you were filling up a cone with water, three things would be changing: the volume of the water, the height/depth of the water, and the radius of the surface of the water. All three would get a variable for the equation that ties all three quantities together
   - If you were filling up a cylinder, only the volume of the cylinder and the height/depth of the cylinder would be changing; the radius WOULD BE CONSTANT!

4. In preparation for the AP Test, review related rates problems involving the Pythagorean Theorem and area/volume problems

TOPIC: *Finding Absolute Max/Min on a CLOSED INTERVAL*

1. Remember, any continuous function on a CLOSED INTERVAL has an Absolute Maximum AND an Absolute Minimum value (**EXTREME VALUE THEOREM**).

2. To find the absolute max/min of $f(x)$ on a CLOSED interval, do the following:
   - Indentify the endpoints – you will need them for a complete justification of your absolute max/min location
   - Find the critical values of $f(x)$. This will require you determining where $f'(x)=0$ or $f'(x)$ does not exist
     - (a) If you have the function for $f'(x)$, then find the critical values using your calculator or algebra skills
     - (b) If you have the graph for $f'(x)$, then look for where $f'(x)$ crosses the *x*-axis (not touches the axis…CROSSES IT!)
   - Create a chart where you test both the endpoints and the critical values using THE ORIGINAL FUNCTION!!!! **(do not plug values into $f'(x)$!!!!!)**
   - **State the absolute maximum/minimum based on the values from the table you computed**

3. In the case where $f(x)$ only has ONE CRITICAL VALUE on the closed interval, and $f'(x)$ CHANGES SIGN at that value, then that value will be the absolute max or absolute min (depending on which type of sign change took place)
   - **Homerun Statement for Absolute Max**: *Since f' changes from positive to negative at $x=c$, and since c is the only critical value on the interval, f will increase until c and then decrease after c* on the entire interval (You must talk about how the function behaves from endpoint to endpoint)
   - **Homerun Statement for Absolute Min**: *Since f' changes from negative to positive at $x=c$, and since c is the only critical value on the interval, f will decrease until c and then increase after c* on the entire interval

TOPIC: *Finding Absolute Max/Min on an OPEN INTERVAL*

1. These cases are usually rare, but they are possible.

2. When these cases do arise, usually there is only ONE CRITICAL VALUE on the interval in question. When this the case, proceed as follows:
   - In the case where $f(x)$ only has ONE CRITICAL VALUE on the closed interval, and $f'(x)$ CHANGES SIGN at that value, then that value will be the absolute max or absolute min (depending on which type of sign change took place)
   - **Homerun Statement for Absolute Max**: *Since f' changes from positive to negative at $x = c$, and since c is the only critical value on the interval, f will increase until c and then decrease after c* on the entire interval (You must talk about how the function behaves from endpoint to endpoint)
   - **Homerun Statement for Absolute Min:** *Since f' changes from negative to positive at x = c, and since c is the only critical value on the interval, f will decrease until c and then increase after c on the entire interval*

3. If there is more than one critical value on the interval, you will need to compare the *y*-values of the original function at each of the critical values (i.e., which one is higher/lower?). You will also need to address what is happening with the function as *x* approaches the 'endpoints' of the interval (when I say 'endpoints,' I mean that you need to look at what will happen to the function as *x* moves towards, for example, positive or negative infinity).

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TOPIC: *Answering Absolute Max/Min Problems Appropriately*

One of the biggest errors that occur in Calculus is the misunderstanding the answer sought for a max/min question. Here is how to approach this issue:

1. If the question asks you to….
   - "Find the maximum value of the function" – STATE THE **Y-VALUE**
   - "Find the minimum value of the function" – STATE THE **Y-VALUE**
   - "Find the absolute maximum value of the function" – STATE THE **Y-VALUE**
   - "Find the absolute minimum value of the function" – STATE THE **Y-VALUE**
   - "Find the location of the maximum value of the function" – STATE THE **X-VALUE**
   - "Find the location of the minimum value of the function" – STATE THE **X-VALUE**
   - "Find where the maximum value of the function occurs" – STATE THE **X-VALUE**
   - "Find where the minimum value of the function occurs" – STATE THE **X-VALUE**

2. If you are not sure, then use the following approach:
   - For Absolute Max: "The absolute maximum value occurs at x = _____, and that maximum value is ________
   - For Absolute Min: "The absolute minimum value occurs at x = _____, and that minimum value is ________

3. Finally, please USE COMPLETE SENTENCES TO STATE YOUR ANSWER. NO LAZINESS. YOU ARE IN VARSITY MATH…WE USE SENTENCES HERE.

TOPIC: *Facts about the 1st Derivative*

For all statements on this card, we will let $f(x)$ serve as our main function to analyze.

1. If $f'(c) = 0$, then $f(x)$ has either a relative max, relative min, or a 'fax machine' at $x = c$
2. If $f'(c)$ does not exist, then $f(x)$ has a cusp, corner point, asymptote, or some discontinuity at $x = c$
3. If $f'(x)$ is positive, then $f(x)$ is increasing
4. If $f'(x)$ is negative, then $f(x)$ is decreasing
5. If $f'(x)$ changes from positive to negative at $x = c$, then $f(x)$ has a relative maximum at $x = c$
6. If $f'(x)$ changes from negative to positive at $x = c$, then $f(x)$ has a relative minimum at $x = c$
7. If $f'(x)$ does not change sign at $x = c$, then $f(x)$ **could** have a 'fax machine,' cusp, asymptote, corner point, etc., at $x = c$
8. If $f'(x)$ has a relative maximum value at $x = c$, then $f''(x)$ will change sign at $x = c$, and $f(x)$ will have a point of inflection at $x = c$
9. If $f'(x)$ has a relative minimum value at $x = c$, then $f''(x)$ will change sign at $x = c$, and $f(x)$ will have a point of inflection at $x = c$

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TOPIC: *Facts about the 2nd Derivative*

For all statements on this card, we will let $f(x)$ serve as our main function to analyze.

1. If $f''(c)=0$, then $f'(x)$ has either a relative max, relative min, or a 'fax machine' at $x = c$
2. If $f''(c)$ does not exist, then $f'(x)$ has a cusp, corner point, asymptote, or some discontinuity at $x = c$
3. If $f''(x)$ is positive, then $f'(x)$ is increasing and $f(x)$ is concave up
4. If $f''(x)$ is negative, then $f'(x)$ is decreasing and $f(x)$ is concave down
5. If $f''(x)$ changes from positive to negative at $x = c$, then $f'(x)$ has a relative maximum at $x = c$
6. If $f''(x)$ changes from negative to positive at $x = c$, then $f'(x)$ has a relative minimum at $x = c$
7. If $f''(x)$ does not change sign at $x = c$, then $f(x)$ **DOES NOT HAVE A POINT OF INFLECTION AT $x = c$**
8. If $f''(c)>0$ and $f'(c)=0$, then $f(x)$ has a relative minimum value at $x = c$
9. If $f''(c)<0$ and $f'(c)=0$, then $f(x)$ has a relative maximum value at $x = c$

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TOPIC: *Justifying Relative Max/Min, Points of Inflection*

For all statements on this card, we will let $f(x)$ serve as our main function to analyze.

When justifying the location of a relative max or a relative min for a function $f(x)$, show/state the following:

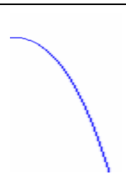
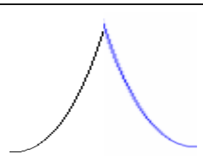
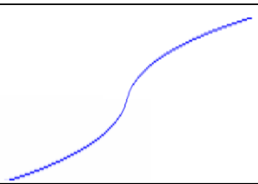
1. Relative Max
   - □ 1st Derivative Test: $f(x)$ has a relative max at $x = c$ since $f'(x)$ changes from positive to negative at $x = c$
   - □ 2nd Derivative Test: $f(x)$ has a relative max at $x = c$ since $f'(c) = 0$ and $f''(c) < 0$

2. Relative Min
   - □ 1st Derivative Test: $f(x)$ has a relative max at $x = c$ since $f'(x)$ changes from negative to positive at $x = c$
   - □ 2nd Derivative Test: $f(x)$ has a relative max at $x = c$ since $f'(c) = 0$ and $f''(c) < 0$

3. Point of Inflection
   - □ 1st Derivative Test: $f(x)$ has a point of inflection at $x = c$ since $f'(x)$ changes from increasing to decreasing or decreasing to increasing at $x = c$
   - □ 2nd Derivative Test: $f(x)$ has a point of inflection at $x = c$ since $f''(x)$ changes sign at $x = c$

TOPIC: *Curve Sketching / Curve Analysis*

For all statements on this card, we will let $f(x)$ serve as our main function to analyze.

Graph Shape	What is *f* doing?	What is $f'(x)$ doing?	What is $f''(x)$ doing?
	Increasing, Concave Down	$f'(x)$ is positive and decreasing	$f''(x)$ is negative
	Decreasing, Concave Up	$f'(x)$ is negative and increasing	$f''(x)$ is positive
	Decreasing, Concave Down	$f'(x)$ is negative and decreasing	$f''(x)$ is negative
	Decreasing, Concave Up	$f'(x)$ is positive and increasing	$f''(x)$ is positive
	Cusp	$f'(x)$ does not exist	$f''(x)$ does not exist
	Vertical Tangent Line	$f'(x)$ does not exist	$f''(x)$ does not exist

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TOPIC: *Tangent Line Approximation*

1. Remember that a tangent line approximation is a tangent line equation used to approximate the value of a curve near the point of tangency

2. To generate a tangent line approximation for a function $f(x)$ at $x = c$, do the following:
   - Find the equation of the tangent line to $f(x)$ at $x = c$
   - Use the tangent line to approximate values of $f(x)$ near $x = c$.
   - For example, the tangent line approximation for $f(x) = 3x^2 - 5x$ at $x = 1$ is $y = x - 3$, which can be found by finding the slope, $f'(1) = 1$, and using the point $(1, -2)$. Now, you can approximate values of $f(x)$ near $x = 1$; for example, to approximate $f(1.02)$, simply plug 1.02 into x in the equation $y = x - 3$ (which would give a result of -1.98)

3. Tangent line approximations will OVER-APPROXIMATE the *y*-values for a function when the function is CONCAVE DOWN (tangent lines will be above the curve)

4. Tangent line approximations will UNDER-APPROXIMATE the *y*-values for a function when the function is CONCAVE UP (tangent lines will be below the curve)

TOPIC: *Mean Value Theorem*

1. The Mean Value Theorem only applies to a function $f(x)$ if $f(x)$ is continuous on the closed interval [*a*, *b*], and differentiable on the open interval (*a, b*)
   - Remember that MVT does not apply on intervals that contain a cusp, corner point, asymptote, etc.
   - Remember also that the endpoints CANNOT be the value for *c*

2. Essentially, the theorem guarantees that $\frac{f(b)-f(a)}{b-a}=f'(c)$. This implies that the average rate of change of $f(x)$ on the interval [*a, b*] must be equal to the instantaneous rate of change of $f(x)$ at some point $x=c$ on the interval.

3. Mean Value Theorem questions tend to start in the following manner:

   Verify that there is some number c on $a<c<b$ such that $f'(c)=k$

   To do that, simply set up the MVT: $\frac{f(b)-f(a)}{b-a}=f'(c)=k$

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TOPIC: *Integral Formulas You Must Know*

1. $\int k\,dx = kx + C$

2. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C$

3. $\int \sin u\,du = -\cos u + C$

4. $\int \cos u\,du = \sin u + C$

5. $\int \tan u\,du = \ln|\sec u| + C$

6. $\int \cot u\,du = \ln|\sin u| + C$

7. $\int \sec u\,du = \ln|\sec u + \tan u| + C$

8. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$

9. $\int \dfrac{1}{u}\,du = \ln|u| + C$

10. $\int e^u\,du = e^u + C$

11. $\int a^u\,du = \dfrac{1}{\ln a} \cdot a^u + C$

12. $\int \dfrac{1}{\sqrt{a^2 - u^2}}\,du = \arcsin\left(\dfrac{u}{a}\right) + C$

13. $\int \dfrac{1}{a^2 + u^2}\,du = \dfrac{1}{a}\arctan\left(\dfrac{u}{a}\right) + C$

14. $\int \dfrac{1}{u\sqrt{u^2 - a^2}}\,du = \dfrac{1}{a}\operatorname{arcsec}\left(\dfrac{|u|}{a}\right) + C$

15. $\int \sec^2 u\,du = \tan u + C$

16. $\int \csc^2 u\,du = -\cot u + C$

17. $\int \sec u \tan u\,du = \sec u + C$

18. $\int \csc u \cot u\,du = -\csc u + C$

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TOPIC: *Riemann Sums*

Riemann Sums are merely approximations of a the values of a definite integral (or area under a curve)

1. Upper Sums: rectangles that OVER-approximate the area under a curve
2. Lower Sums: rectangles that UNDER-approximate the area under a curve

3. Left-Hand Sums: multiply (length of the interval) x (left-hand *y*-value for the interval); add up the areas of all necessary rectangles
   - PAY ATTENTION THE LENGTHS OF THE INTERVALS (are they equal or not?)

4. Right-Hand Sums: multiply (length of the interval) x (right-hand *y*-value for the interval); add up the areas of all necessary rectangles
   - PAY ATTENTION THE LENGTHS OF THE INTERVALS (are they equal or not?)

5. Midpoint Sums: multiply (length of the interval) x (midpoint *y*-value for the interval); add up the areas of all necessary rectangles
   - INTERVALS MUST BE EQUAL, AND YOU NEED THE MIDDLE *x and y* VALUES FOR EACH INTERVAL TO DO THIS

6. Trapezoidal Sums: do not worry about memorizing the actual Trapezoidal Rule. Just use the formula for the area of a trapezoid to find the area for each subinterval; $A = \frac{1}{2}(y_1 + y_2) \cdot \Delta x$

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TOPIC: *Sigma Stuff*

1. Sigma Formulas you may want to recall: $\sum_{i=1}^{n} c = c \cdot n$, $\sum_{i=1}^{n} i = \frac{n \cdot (n+1)}{2}$, $\sum_{i=1}^{n} i^2 = \frac{n \cdot (n+1) \cdot (2n+1)}{6}$

2. Recall that the definition of a definite integral comes from the following sum:
   - $\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x \quad \Rightarrow \quad \int_a^b f(x)\, dx$

3. Remember how to determine a definite integral from a sum. For example:
   - $\lim_{n \to \infty} \frac{4}{n} \left[\left(\frac{3n+4}{n} \right)^3 + \left(\frac{3n+8}{n} \right)^3 + \left(\frac{3n+12}{n} \right)^3 + \ldots + \left(\frac{3n+4n}{n} \right)^3 \right]$
   - First, use the $\frac{4}{n}$ to determine the length of the interval ($b - a = 4$ in this case)
   - Look at the last term in the sum and simplify it: $\frac{3n+4n}{n} = 7$
   - Now determine the lower limit of integration (in this case, it would be 3 since the interval ends at 7, and is 4 units long)
   - Finally, determine the function that will be integrated (basically, what's happening to each of the terms in the sum?)
   - Final Answer: $\lim_{n \to \infty} \frac{4}{n} \left[\left(\frac{3n+4}{n} \right)^3 + \left(\frac{3n+8}{n} \right)^3 + \left(\frac{3n+12}{n} \right)^3 + \ldots + \left(\frac{3n+4n}{n} \right)^3 \right] = \int_3^7 x^3\, dx$

TOPIC: *Fundamental Theorem of Calculus, AP STUNNERS*

1. First Form: $\int_a^b f'(x)\,dx = f(b) - f(a)$
   - **Remember that this computes the CHANGE in $f(x)$ on the interval from *a* to *b***

2. Second Form: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$
   - Remember that this rule applies when you are taking the derivative of an integral
   - Remember to use the chain rule when the limits of integration are functions of x:

     $\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\,dt = f(h(x))\cdot h'(x) - f(g(x))\cdot g'(x)$

3. AP STUNNERS
   - In an AP STUNNER situation, you are given a 'rate' function and some initial condition about that function from which the 'rate' originated.
   - Essentially you are going to set up the 1st Form of the Fundamental Theorem,

     $\int_a^b f'(x)\,dx = f(b) - f(a)$
   - For example, suppose you knew the velocity for an object, $v(t)$, and you knew that the position at time $t = 0$ was $x(0) = 4$. If asked for the position at $t = 5$, simply set up and solve the following integral: $\int_0^5 v(t)\,dt = x(5) - x(0) \quad \Rightarrow \quad x(5) = x(0) + \int_0^5 v(t)\,dt$

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TOPIC: *Substitution with Integration*

1. Remember that we only have 4 basic known forms of integrals for this course:
   - $\int u^n \, du$, $\int \frac{1}{u} \, du$, $\int e^u \, du$, $\int trig \, du$

2. If the integral is not in one of those forms, you must use substitution or integral property to get the integrand of the integral into one of those known forms.

3. Remember to apply the rules of LIATE (logs, inverse trig, algebraic, trig, and exponential function) when picking the "*u*"*;* the pecking order for choosing the *u* is based on the order of the letters in LIATE.

4. CHET LOI – do not forget to CHANGE THE LIMITS OF INTEGRATION when evaluating definite integrals with substitution

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TOPIC: *Exponential Growth and Decay*

1. When solving the differential equation $\frac{dy}{dt} = ky$, remember the resulting solution will be $y = y_0 \cdot e^{kt}$ (this results form solving the differential equation using separation of variables)

2. The differential equation $\frac{dy}{dt} = ky$ is verbally translated as "the rate of change of y is proportional to y"

TOPIC: *Solving Differential Equations*

1. Any differential equation is an equation involving a derivative or differential term

2. The solution to a differential equation occurs when you have eliminated the differential terms and gotten an equation in terms of x and y.

3. To solve a differential equation, remember to do the following:

   - □ SEPARATE THE VARIABLES!!!
   - □ Integrate both sides of the equation
   - □ Include the + C on one side
   - □ If an initial condition is given, solve for the C.
   - □ Solve for y if possible

Please be careful with differential equations where you have terms like $\frac{dy}{y^n}$ …the resulting antiderivative is NOT $\ln\left|y^n\right|$; pay attention to the structure of the integral and use the appropriate integration rules

TOPIC: *Area between 2 Curves*

1. To find the area between two curves, use the formula: $A=\int_a^b \left(f(x)-g(x)\right)dx$, where $f(x)\geq g(x)$ (essentially, integral of the top curve minus the bottom curve)

2. Remember to pay attention to where the region is defined

3. You may have to find the locations of where the area is defined if not provided (i.e., set the equation equal to each other, etc.)

4. Pay careful attention to regions where the top and bottom curve change/switch which one is located above the other; if they switch, you will need to set up two integrals. The area should always be computed using a (top curve – bottom curve) approach.

TOPIC: *Volume*

1. Disks: $V = \pi \int_a^b (r(x))^2\, dx$; usually $r(x) =$ height of the curve off of the *x*-axis
   - □ Used mostly when rotating around the *x*-axis or a horizontal line
   - □ If rotating around *y*-axis or vertical line, you must change integral to be in terms of *y*

2. Washers: $V = \pi \int_a^b \left[(R(x))^2 - (r(x))^2 \right] dx$; $R(x) =$ outer radius, and $r(x) =$ inner radius
   - □ Used mostly when rotating around the *x*-axis or a horizontal line
   - □ If rotating around *y*-axis or vertical line, you must change integral to be in terms of *y*

3. Shells: $V = 2\pi \int_a^b r(x) \cdot h(x)\, dx$; $r(x) =$ shell radius, and $h(x) =$ shell height
   - □ Used mostly when rotating around the *y*-axis or a vertical line
   - □ If rotating around *x*-axis or horizontal line, you must change integral to be in terms of *y*

4. Known Cross-Sections: $V = \int_a^b A(x)\, dx$, where $A(x) =$ area of the cross-section
   - □ If cross-sections are perpendicular to *x*-axis, integrate with respect to *x*
   - □ If cross-sections are perpendicular to *y*-axis, integrate with respect to *y*

5. When rotating around non-axis lines (for example, around $y = 2$ or $x = 4$), carefully set up your radius function (the rule of thumb is always top – bottom or right – left…whether that be the rotating axis or curve being rotated)

TOPIC: *Position, Velocity, Acceleration*

1. Basic Relationships
   - $x'(t) = v(t)$; $x''(t) = v'(t) = a(t)$

2. An object moves to the right/up when $v(t) > 0$, and it moves to the left/down when $v(t) < 0$

3. An object is at rest when $v(t) = 0$

4. An object changes direction when $v(t)$ changes sign

5. An object speeds up (increases speed) when velocity and acceleration agree in sign; an object slows down (decreases speed) when velocity and acceleration disagree in sign

6. An object moves away the origin when position and velocity agree in sign; an object moves towards the origin when position and velocity disagree in sign

7. Displacement = $x(b) - x(a)$

8. Total Distance an Object Travels = $\int_a^b |v(t)|\, dt$

TOPIC: *Rate In, Rate Out Questions*

1. In these questions, you are usually provided with a function modeling the rate at which something goes in to something, and the you are usually provided with a function modeling the rate at which something goes out of something (e.g., the rate at which water flows into a bath tub, and the rate at which water flows out of the drain).

2. You may be given some initial condition about the situation (such as how much water was in a tub when you started the faucet, etc.)

3. If you integrate the 'rate-in' function, you will compute the **CHANGE** in the quantity that goes in (e.g., if you integrate the rate of flow of water into the tub, the integral will compute the **change** in the amount of water that flowed into the tub)

4. Many times you will be asked to generate a function which computes the amount/quantity present (i.e., the 'accumulation function'). To generate this, you will use this basic structure:
   - Current Amount = Initial Amount + $\int_a^b (rate\ in) - (rate\ out)\, dt$
   - This is essentially a manipulation of the AP Stunner

5. You may also be asked to compute an absolute maximum or minimum value; remember that optimal situation occurs whenever the (rate-in) = (rate-out).