Continuity, Intermediate Value Theorem, Secant and Tangent Lines

UCLA AP Readiness Sept 27, 2014 Presenter: Stephen Lange slange@lausd.net

Recommended Websites

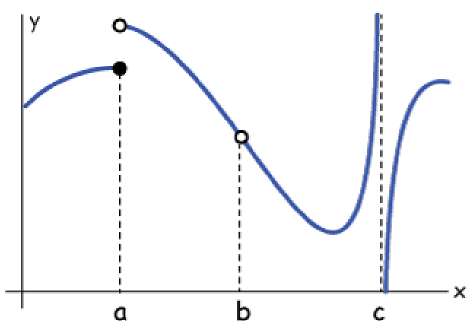
• Khanacademy.org - videos/exercises	• Wolframalpha.com- solves it all
• Calculusapplets.com – active learning	• Mastermathmentor.com – good materials for all
• Desmos.com – best online graphing calculator	• Patrickjmt.com- great videos

Continuity

Continuity of a function at a point

The function $f(x)$ is continuous at a point x_0 in its domain if

$$\lim_{x \to x_0} f(x) = f(x_0)$$



In this case it's more instructive to look at the limit relationship above in terms of discontinuities and see how it fails.

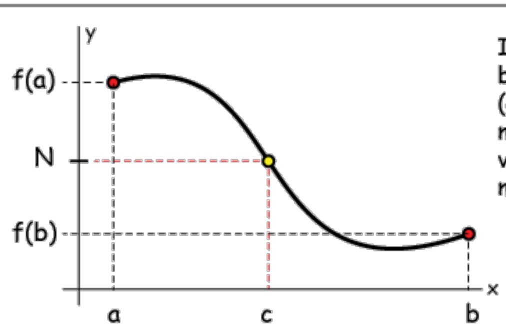
In the case of a step discontinuity (a), the left and right limits are different, so the limit does not exist (DNE). At the removable discontinuity (b), f(b) does not exist; although we can get very close to b, we can't actually get there. At (c) the limit is undefined: from the left it's $+\infty$ and from the right it's $-\infty$.

1. What does it mean for a function to be continuous in simple terms?
2. What are some examples of discontinuities?
3. What are functions that are continuous?
4. What are functions that have discontinuities?
5. What is the formal definition of continuity?

The Intermediate Value Theorem

The Intermediate Value Theorem

If f is a continuous function on the interval $[a, b]$, and N is any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$, then there is a number c in (a, b) such that $f(c) = N$.



If the graph of a function can be traced continuously from $(a, f(a))$ to $(b, f(b))$, then it must take on every intermediate value from $f(a)$ to $f(b)$ —possibly more than once — along the way.

How does the intermediate value theorem help us find zeroes/roots?

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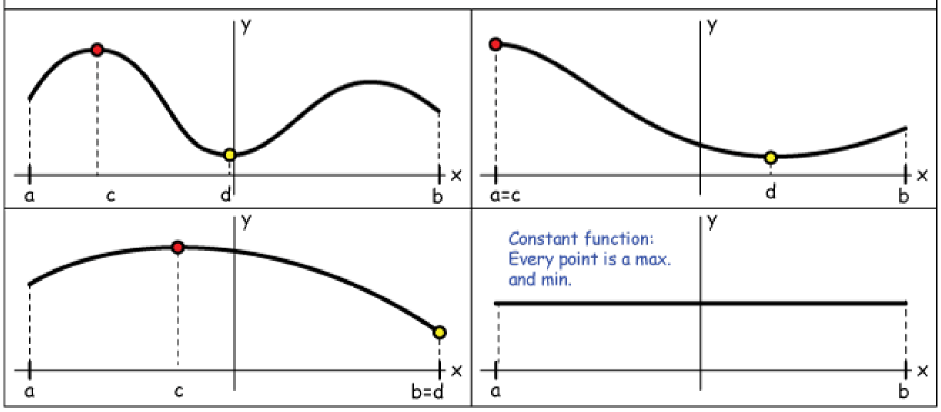
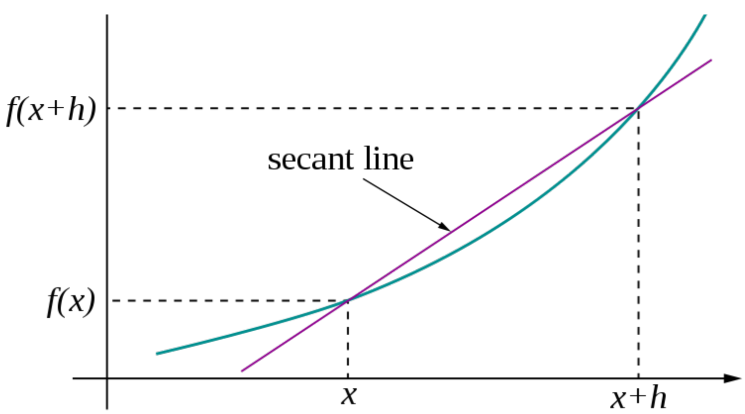
Extreme Value Theorem

The Extreme Value Theorem
If f is a continuous function on the closed interval [a, b], then f has both an absolute maximum (●) value, f(c), and an absolute minimum (●) value, f(d), for some numbers c and d on [a, b].



Secant Lines

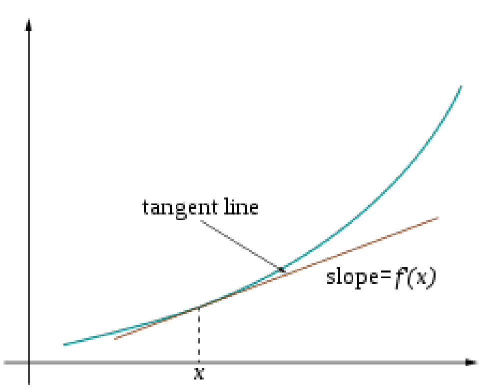


$$A.R.C. = \frac{\text{change in } f(x)}{\text{change in } x} = \frac{f(x) - f(a)}{x - a}$$

$$\frac{f(x+h) - f(x)}{h}$$

$$slope = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(t_1) - f(t_0)}{t_1 - t_0} = Average.Rate.ofChange$$

Tangent Lines



$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$